DEVELOPMENT OF METHODOLOGY FOR HIGHWAY AGENCIES TO FOLLOW IN ESTABLISHING OPTIMUM ACCEPTANCE PLANS FOR PAVEMENTS

By

YVES JOSEPH ANGLADE

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dedicated to my mother

Mrs. Denise Janvier

and

to the memory of my father

Mr. Joseph Mathieu

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KEY TO SYMBOLS

English & Greek Symbols

μ_0	Target value of the variable quality characteristic to be controlled
x	Deviation of the quality characteristic of an individual unit from the target value
μ	Deviation of the mean of the quality characteristic in a given inspection lot from the target
σ^2	Variance of x in a given inspection lot
N	Number of units contained in a given lot
n	Sample size
Φ	Cumulative standard normal function
α	Consumer's risk
β	Producer s risk
$\bar{x}$	Sample mean
S_L	Lower specification limit
S_U	Upper specification limit
L	Lower acceptance limit for x
U	Upper acceptance limit for x
$f(x \mid \mu)$	Probability density function of x, given that the deviation of the mean of the inspection lot from μ_0 is μ
$g(x \mid \mu)$	Probability density function of X, given that the deviation of the mean of the inspection lot from is
$h(\mu)$	Probability density function of the lot mean μ

σ_μ^2	Variance of μ
C_s	Constant cost of preparing the inspection
C_i	Variable sampling and inspection cost per unit
C_r	Rejection cost per unit
ψ	Normal density function with mean 0 and variance $\sigma^2(n+D)/nD$, independent of μ

Abstract of Dissertation Presented to the Graduate School
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DEVELOPMENT OF METHODOLOGY FOR HIGHWAY AGENCIES TO FOLLOW IN ESTABLISHING OPTIMUM ACCEPTANCE PLANS FOR PAVEMENTS

By

Yves J. Anglade

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Major Department: Civil Engineering

The objective of this research is to develop an approach for establishing, optimum pavement and bridge construction acceptance plans that minimize the overall life cycle cost. To this end, an understanding of theoretical principles underlying statistical acceptance-sampling plans and current practices is essential.

In this research, emphasis is placed on developing a methodology for the determination of the optimum sample size (i.e., number of samples) in acceptance sampling using a Bayesian statistic approach. A framework is established to integrate the many aspects that deal with each phase of the pavement life cycle in order to design, construct, and maintain the pavement at a minimum life cycle cost.

Operating characteristics curves are used to evaluate the risk associated with optimum sample size derived from the mathematical-statistical cost model. As part of the comprehensive framework, significant attention is given to the development of guidelines for the evaluation of quality characteristics and tolerances using Genichi Taguchi methods.

Finally, a pay adjustment method based on the quadratic loss function is developed to compensate for loss in quality resulting from the product's failure to meet the target design value.

To illustrate this approach, estimated sampling cost and life cycle cost of pavement are used to determine the optimum sample size and the acceptance limits for the example. Boundary conditions are formulated and used to perform a sensitivity analysis. a discussion on the performance of the model as compared to statistically based models is also provided

CHAPTER 1
PROBLEM STATEMENT AND RESEARCH OBJECTIVES

Introduction

The state of surface transportation infrastructure in the United States is one of the most important challenges of current time. A large number of bridges and highways have either deteriorated or become obsolete, and a continuing increase in population poses a new demand for improved roads, highways, and bridges.

It is estimated that with the current surface transportation condition, $62.7 billion will be required to maintain the present condition and $86.9 billion if the condition is expected to improve. In 1993, an estimated 49.7 billion was projected to simply maintain the overall condition and performance on arterial, collectors, and local highways [Transportation and Environmental Infrastructure Needs 1994 pp. 225-226]. The fact that some of the roads and highways in place today have deteriorated prematurely leads many experts to believe that, among the many possible causes for this occurrence construction materials variability and inadequate quality control may be the greatest contributors.

If pavements and bridges are designed only for a given design life, their manufacturing and maintenance costs may not be optimal. There is a growing need to design pavements and bridges so as to minimize overall life cycle costs rather than just a segment of the life cycle costs such as manufacturing costs. Designing a pavement or a bridge in terms of its life cycle costs ensures an overall economy and not the economy of the purchase price only Also in many industries it has been recognized that the greatest impact on overall life cycle costs results from the design decisions made at the early stages of product design For example, the material specifications or allocation of tolerances and acceptance criteria at the design stage can have great impact on the cost of manufacturing, and on the quality, reliability, and life of the product.

In December of 1991 U.S Congress enacted a landmark surface transportation legislation, the Intermodal Surface Transportation Efficiency Act (ISTEA) It reaffirmed the U S Department of Transportation's (USDOT) commitment to an intermodal transportation system by providing increased funding levels and program flexibility Recognizing that the transportation decisions dramatically affect the country's economy, global competitiveness and environment as well as the quality of life in, the ISTEA was designed to assist agencies in making better transportation related decisions One of its requirements imposes on transportation agencies the consideration of life cycle cost in their projects. The Intermodal Surface Transportation Efficiency Act [1991]

1.2 Problem Statement

The existence of variation in highway construction materials, products, and construction methods remains a major challenge for transportation agencies. The absence of a sound method of controlling inherent variations in the construction process can have disastrous consequences such as premature deterioration of products and high cost of maintenance or replacement

Back in 1979, the Transportation Research Board/Federal Highway Administration (TRB/FHWA) sponsored a series of workshops to identify research and development needs related to highway construction engineering management. Participants included representatives from the highway industry, state highway agencies, related federal agencies, contractors, universities, and consultants.

The workshop defined 63 R&D needs, of which "Cost Effectiveness-- Sampling and Testing" was identified as the number one priority. Since 1979, some research into the cost effectiveness of sampling and testing has been conducted, but the research has not been generally successful. The quality assurance testing programs are estimated to cost approximately $200 million per year for federal aid highway projects alone. Von Quintus et al [1985]. According to an FDOT official, the State of Florida Department of Transportation estimates that quality control cost ranges from 10 to 15 percent of the contract price.

Currently, most transportation agencies, including FHWA, use statistically based quality assurance programs to address the issue of product control and acceptance. Apparently, these programs are inadequate to respond to cost-effectiveness concerns expressed by transportation agencies. Some aspects of sampling and testing method are arbitrarily selected. For example, most state transportation agencies employ statistically based acceptance plans that call for 5 random samples to be tested per lot; these agencies use n=5 no matter what the quality characteristic (e.g., strength or thickness) is being tested. A recently developed prototype performance related specification (PRS), however, calls for 10 random samples (actually five averages of 2 random samples), again no matter what the quality characteristic. Evidence exists that n=5 is usually too small to estimate the quality with sufficient confidence to enable determination of accurate and fair pay adjustment. The optimum number of samples should be based primarily on the cost of the test to be performed, the variability of the quality characteristic being measured, and the risk that state highway agencies and contractors are willing to take. This risk involves the cost of making a wrong acceptance decision. Wrong acceptance decisions mean accepting poor quality concrete, rejecting good quality concrete, or assigning a wrong pay adjustment for concrete quality. It should be obvious that since the costs of various tests are different, and the variability associated with different quality characteristics are different, the optimum number of samples should not always be n=5 or n=10 for the same assumed risk.

There exists a need for a convincing method of analysis defining the most cost-effective, or optimum, number of samples and tests, acceptance limits, and the consequences associated with an assumed risk.

In keeping with the tradition of transportation agencies to give priority to providing services to the public, the proposed research will develop a much needed methodology to assist them in their decision making process. In the development of such a tool, three key items, which are considered most critical in the effort to remedy or greatly improve current conditions, are highlighted: optimum sample size, frequency of test samples, and consequences of risk. The resulting optimum plan is the one that minimizes the life cycle cost of pavement.

1.3 Research Objectives

The proposed research to develop a methodology for establishing optimum acceptance plans is invaluable to federal and state transportation agencies. As more knowledge of construction materials, properties, and their behavior are acquired, more and more economic constraints are being imposed on agencies to do more with less, further justifying the need for a method capable of improving the agencies' ability to manage systems effectively and efficiently. Therefore, practicality, applicability, and implementation of the proposed methodology underlie the objectives of this project. The resulting report should benefit not only the state of Florida but the Federal Highway Administration and other states as well.

As stated previously, the objective of this research is to develop a methodology to be employed by transportation agencies for designing optimum acceptance plans for pavements, particularly concrete pavements. Even though the model presented in this research was intended for concrete pavement and the use of strength as an example, it is very general in its range of application to other materials and quality characteristics. The proposed optimization methodology will incorporate the cost effectiveness of sampling and the associated sampling frequencies used in controlling the quality of pavement materials as related to pavement performance, acceptance limits, and risk associated with a particular decision. Simply put, the optimization model will consider the cost of testing for a particular quality characteristic, its variability, and its relative impact on the life cycle of pavement.

An optimum acceptance sampling cost model by itself can only be of limited use to transportation agencies. Its full effectiveness is only achieved insofar as all other aspects of the product are optimized. Hence, a further objective of this research is to develop a comprehensive framework in which the primary objective can produce optimal results.

The comprehensive framework referenced above defines an integrated approach where all the elements of the optimum performance of pavement are planned and analyzed at the design stage while focusing on their interrelationships, that is, optimal design, tolerances, maintenance cost, and acceptance sampling. The latter constitutes the bulk of this research.

The integrated approach allows for a product's life cycle cost to be determined in terms of its optimum cost per hour, day, or year of operation. Life cycle cost consists of manufacturing cost, internal and external quality cost, and operating and maintenance cost during the life of the product. These three components of the life cycle cost are strongly influenced by not only the parameters of the product design, but also the tolerances for those parameters of the product design.

In order to achieve the stated objectives, the specific tasks outlined below were found to be essential,

1. To develop a statistical and mathematical cost model to determine the optimum sample number of acceptance samples.
2. To develop a method for establishing tolerances for each quality characteristic under consideration,
3. To incorporate in the methodology a means for risk analysis;
4. To develop a rational pay adjustment schedule based on the quality lost to society resulting from product deviation from target value.

5. To develop guidelines for transportation agencies to determine and evaluate appropriate quality characteristics
6. To develop a methodology for sensitivity analysis of optimum sample size with respect to life cycle decision parameters and active constraints.

It is not within the scope of this research to collect actual data from transportation agencies for the purpose of determining the exact sample size for the acceptance sampling of concrete pavement The focus is on the development of methodology However, reasonable data is used for illustrative purposes

1.4 Research Approach

The basic organization of this dissertation is as follows

1.4.1 Phase 1

Phase one starts with background information followed with a definition of the research problem and objectives to complete chapter one The next two chapters present a comprehensive literature search While chapter two is centered on providing general state of the art information related to the research topic, chapter three focuses on establishing an understanding of the essential methods and principles underlying acceptance sampling.

To include the development of the quality cost model based on the methods of the quadratic loss function approach.

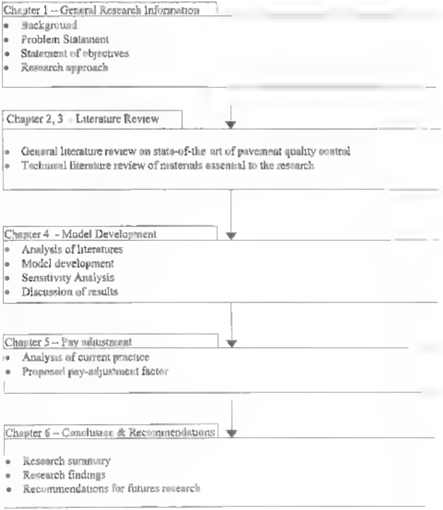

Chapter 1 – General Research Information
Background
Problem Statement
Statement of objectives
Research approach
Chapter 2, 3 – Literature Review
General literature review on state-of-the art of pavement quality control
Technical literature review of materials essential to the research
Chapter 4 - Model Development
Analysis of literatures
Model development
Sensitivity Analysis
Discussion of results
Chapter 5 – Pay adjustment
Analysis of current practice
Proposed pay-adjustment factor
Chapter 6 – Conclusions & Recommendations
Research summary
Research findings
Recommendations for futures research

Figure 1 Research Flow Chart

1.4.2 Phase 2

In chapter four the cost function developed in chapter three for selected quality characteristics is used to develop the optimum acceptance-sampling model. In so doing, a decision theoretic approach is taken where bayesian statistical methods are greatly exploited. An optimization method precedes a sensitivity analysis of the model. An example and discussion of results concludes this phase.

1.4.3 Phase 3

Chapter 5 presents an analysis of a procedure for pay adjustment currently under research. The phase is completed with a proposed adjusted payment method using the quadratic quality loss function.

1.4.4 Phase 4

Chapter 6 presents the summary of the work and finding it concludes with recommendations for future research.

CHAPTER 2
RELEVANT LITERATURE

Although there has been considerable interest in recent times to improve the quality of pavement in all its aspects, the development of a methodology, which integrates all aspects of the design, manufacture, quality assurance, operation, and maintenance in terms of life cycle cost, has not received adequate attention.

2.1 Variability in Highway Products

The quality of highways has always been a major concern to highway engineers and contractors. The AASHTO Quality Assurance Guide to Specifications developed by the American Association of State Highway and Transportation Officials (AASHTO) uses the variability of materials and construction processes as one of the measures to assess quality. This is noted in NCHRP Synthesis 232 [1996]. Also in the synthesis of information prepared for the Transportation Research Board, several measures of variability are defined, but the synthesis concentrates on the use of standard deviation as the usual variability measure. The synthesis up dates typical variabilities found in material and construction processes. Also included are discussions of current research activities as related to variability: how variability can be used in the development of specification limits, the state-of the-practice of incentives and disincentives in specifications, and the need for additional information on the variability of several material and construction processes.

Literature also reveals that of the many important findings from the AASHTO Road Test conducted in 1962, one was a revelation as to the greater than expected magnitude of the variabilities encountered in the construction of pavements. Since the Road Test findings were reported, the FHWA and various state department of transportation (DOTs) have conducted many studies on typical variability in highway construction. More recently, with the increased use of the computer to accumulate the data from materials quality control and acceptance testing, it has also been found that both DOTs and contractors have developed data bases for quantifying material and construction variability as well as other important properties.

According to the TRB synthesis, data collected in past research reveals that variability has a relatively wide range of values for each test procedure. It is cited that some of the factors that influence this variability are: the period of time, distance, area, quantity of material, etc., over which the variability is measured. This information was not always available in the references. In the synthesis conclusions, the following recommendation was highlighted: in the development of a specification, the collected data should be used prudently and verification of the variability under conditions of proposed usage is encouraged.

The same synthesis points out an important fact, that variability is intuitively well recognized—nothing is constant, everything changes. In nature as well as in manufacturing processes, variability is expected. For example, in the production of concrete, the size of aggregates, weight of cement, air content, and water quantity are among properties that vary between batches within a narrow range. In all aspects of life change is the norm. In fact, the intuitive recognition of variability is often better understood than the definition found in a dictionary, which is "apt or liable to vary or change."

The synthesis also notes that variability as used in that work is the quantification of typical variation found in materials and construction processes in highway pavement construction. The key word in this usage is "typical." Typical is a relative term. It may or may not apply to the variability of a process that targets a particular specification depending on the sources of variability operating in the acceptance plan for that specification. It is important to recognize that sources of variability are present and that these sources appreciably influence the magnitude of variability found.

Although variability is well recognized intuitively, literature supports the fact that routinely, most engineers are surprised when two test results on engineering properties from the same sample are not the same. This often leads to the question, how much variation can be reasonably expected from two test values from the same material? Also, the common assumption is usually the following: if the two values differ in any way, one of them must be in error. Clearly, a need has been identified to somehow relate variability and acceptance limits, and this is one of the many concerns of this research.

Another interesting finding is that the importance of the magnitude of variability has been recognized in the highway industry since at least the early 1950s, but it was not until the analysis of the variability of materials and construction from the AASHTO Road

Test (1956-1962) that the magnitude and the effect of the variability on specifications limits made a substantial impact. The analysis found that virtually no materials or construction properties met the specifications 100 percent of the time and some met the specifications less than 50 percent of the time. The synthesis author stated that AASHTO Road Test specifications were intended to represent typical specifications, the kind used every day for control of our large highway construction program." The development of the specifications was through the use of engineering judgment as to what a panel of experts thought could typically be built. A conclusion by researchers Carey and Shook [1966] emanating from the Road Test was:

> Briefly summarizing, we want to show that with many more well-trained inspectors than could economically be used in normal construction, with high-speed testing techniques, with a large-scale materials laboratory on site, with the ability to control in detail the contractor's construction procedures, with a highly competent and cooperative contractor who was well paid for everything he was required to do, and the eyes of the highway fraternity on the back of our necks, we were still unable to meet the specifications for many of the construction items within a country mile (p. 110).

While this conclusion is ancient history to many, the literature search found that, of many specification limits used today, few have little more basis than those discussed by Carey and Shook [1966] that were used in the AASHTO Road Test. Many present day specification limits consider neither process capability nor the performance measures necessary to achieve an adequate product.

However, the information gained from the AASHTO Road Test data analysis led to an appreciable research effort in the 1960s by the Bureau of Public Roads (BPR) and several State Departments of Transportation. This effort led to several notable national conferences and research reports, which proved invaluable in developing magnitudes of variability for typical construction projects.

There is a rather unique partnership termed the National Quality Initiative (NQI) that was formed by AASHTO and FHWA and various industry associations to focus on continuous quality improvement within the highway industry, National Quality Initiative [1994]. Under the guidance of the NQI, the members of the NQI Steering Committee developed a "National Policy on the Quality of Highways." One of the principles of this national policy is a continuing commitment for quality products, information, and service through proper design, construction specifications related to performance, adherence to specifications, use of quality materials, use of qualified personnel, and sufficient maintenance.

Clearly, the use of proper designs and construction specifications related to performance, as well as determining adherence to specifications require knowledge and application of variability.

In June of 1994, the NQI developed a long range plan to help ensure that its objectives on the national policy on the quality of highways are met.

2.2 Statistical Quality Assurance

According to the AASHTO Quality Assurance Guide Specifications [1995], specification limits used to control and accept a product are most appropriate when they are related to a measure of the variability typically found in the process that produces that product particularly if the limits are used in a Statistical Quality Assurance (SQA) specification

The definition of an SQA specification according to the AASHTO Quality Assurance Guide Specifications includes all those planned and systematic actions necessary to provide adequate confidence that a product or service will satisfy given requirements for quality This definition furthermore contains, in general at least two parts a quality control function defined as the sum total of activities performed by the seller (producer manufacturer, and/or contractor) to make sure that a product meets contract specification requirements, and an acceptance program, defined as all factors that comprise the agency's determination of the degree of compliance which contain, among other ingredients both quality control and acceptance limits that are based on a function of the expected, or allowable, variability of the particular process SQA will be discussed in more detail in the coming sections of this research

One of the major advances was made in the 1960s when great emphasis was placed on the use SQA specifications At that time the use of these types of specifications was not completely understood by the construction community Over the last several years engineers have begun to understand SQA more fully and it is now widely used by many states and as mentioned, is being emphasized in the AASHTO Quality Assurance Guide Specification, the NQI and various FHWA and DOT initiatives

In 1969 FHWA's Public Roads printed a six-part article which, among other things, summarized results from research studies that measured the typical variability associated with highway materials and construction, and the components of such variability, NCHRP Synthesis 232 [1996]. These research studies confirmed the presence of the large magnitude of variability that was first revealed to the highway community in the AASHTO Road Test a decade earlier.

As mentioned previously in this chapter, several studies of typical variability were undertaken and reported in the 1960's on soils, asphalt concrete, Portland cement concrete, and properties of pavement layers constructed with these materials. The Public Roads article provides a perspective on not only the materials and construction activities investigated but also the wide geographical nature of the studies. In the intervening years, several additional studies, including the TRB synthesis have been reported. It is generally agreed that because of improvements in technology, process capability, testing equipment, procedures, and of significant importance, an increase in the recognition of the existence of variability, the variability of some processes has decreased. There is also concern, as expressed earlier, that possibly offsetting these positive effects are the negative effects brought about by the loss of experienced personnel which can result in an increase in variability of some products possibly because of the lack of experience or training in sampling and testing techniques.

The literature clearly indicates that the highway construction industry requires proper answers to the following three questions:

(1) What is needed?

(2) How can it be ordered?

(3) How can the resulting product be evaluated to determine if that which was ordered was in fact received?

Answers to the first question encompass the total body of research, development, engineering technology, and experience. All these combine to define needs with respect to material, properties, and design characteristics of the highway component.

Answers to the second question depend on the manner in which the details are spelled out in specifications, specific characteristics that must be controlled, needs with respect to qualitative level, and uniformity of the product from item to item.

Answers to the third question depend on the precision and accuracy of test methods as well as on the time required to perform these tests. Testing time often controls the number of measurements that can be made available for use in decision making. More importantly, the relation of the characteristic, or property, measured by the test to the performance of the completed component is a major consideration, which often is known only empirically, if at all.

2.3 Traditional Quality Assurance

Some of the specifications still in use today in state highway construction can be classified as recipes rather than specifications. They spell out in detail the operations of the contractor, the equipment he must use, and the desired end product he must produce. These traditional specifications have come about because adequate quality definitions and test methods pertaining to the quality of the end product are lacking. When specifications do attempt to define required quality, the specified values for characteristics are often those that are obtained through judgment and experience. Tolerances for such characteristics seldom reflect the true needs and capabilities of the construction process or of the available materials.

Many engineers believe that when traditional specifications are combined with the skills of engineers, the complete cooperation of contractors, and the desire of everyone to do a good job, there is no doubt that a good highway can be built. However, inspectors and engineers must be capable of recognizing good materials and construction, without relying solely on quality measurements. Under most of the present procedures, one periodic sample is taken. This sample is assumed to be representative of the material or construction which is tested, and the test result is recorded as the value of the measured property or characteristic. If the test result is within the stated tolerances, the material passes and is accepted. If the test result is not within the stated tolerances, the material or construction fails to pass. In the latter case, engineering judgement must then be applied and a decision made as to whether the material should be re-tested or whether it may be said to substantially comply because the specification deviation will cause little impairment of performance. It should be noted that the researcher, later in this dissertation, strongly discourages this approach.

Even though a quality assurance system that is based on engineering judgment is workable under proper conditions, the practice is difficult to define in legal or contractual terms. Substantial compliance has not been quantitatively defined, and the degree of acceptable variation will differ from engineer to engineer and from job to job. Sampling and testing errors are often so large that the true variations of the materials or construction may be obscured. Some tests may not truly measure quality of the finished highway.

Improvement in quality assurance of highway construction accordingly entails:

- Development of realistic quality criteria.
- Development of valid quality tests.

- Development of valid decision making rules

With this realization many states have progressively moved to the use of statistical concepts, which are judged to be the most promising tools for the solution of many quality-assurance problems in highway construction. Other industries have been using statistical concepts in process control and acceptance for many years. In fact, the Department of Defense in its procurement program during World War II pioneered much of the development in this field. Because of the nature of the highway industry, some of the methods had to be modified, but the concepts are basic to any industry.

The highway agencies involved in this new endeavor have discovered that the science of statistics is a versatile tool. In situations requiring decisions concerning contractual items that are based on samples, statistical concepts allow various acceptable solutions. Rules for each decision must be carefully defined and followed, but different rules can be formulated for each of the many conditions encountered. Decisions can be made with an established degree of confidence. The degree of confidence required for each decision can be correlated with the criticalness of the decision to the quality of the end product, and the rules can be formulated accordingly.

Test methods are continually being developed for better and more rapid measurement of quality. The greatest advance in new methods of testing has been in the nuclear field. The nuclear moisture-density gauge has been proved to be a fast, accurate method of measuring the moisture and density of compacted materials. Nuclear methods of measuring density and asphalt content of bituminous pavements are showing considerable promise. Seismic methods of measuring compaction are also being developed. Sonic equipment is being used to test welds, and sonic methods of measuring the moduli of concrete have been in use for several years, although not widely accepted. Electronic equipment, using the principles of resistivity and magnetism, has been developed to check the placement of steel in concrete and to measure the thickness of pavement components. Rapid nondestructive tests such as those cited will provide better quality control and make quality measurements available in the future.

Through the work of its different committees, the American Society of Testing & Materials (ASTM) is advancing the state of the art of quality measurement by developing precision statements for standard tests. These statements will provide a basis for evaluating the work of inspectors and laboratory technicians and should decrease testing errors.

Other aids to better achieve quality products are automated processing plants with direct output printout. These plants provide not only automatic control, but also adequate documentation to check output for pay quantities. However, automated control does not guarantee product quality.

2.4 Current Trends in Pavement Quality Control

The literature search has identified two dominant philosophies in the pavement quality control arena. An overview of each is provided. As pointed out in an earlier section, there is hardly an area in the scientific community that has not been touched by statistics. Statistical analysis is a tool by which we make sense of data. Increasingly, state and transportation agencies are relying on statistics to assure quality highway construction. In a nut shell, the goal, again, is to make sure that what is specified is in fact received.

The first philosophy, known as SQA, has already been mentioned. SQA is a distinct departure from the "cookbook recipe" specifications under which contractors were told precisely how to perform each step of construction. Instead, SQA gives contractors considerable latitude to use their expertise and ingenuity, and relies on statistical and mathematical concepts to determine if the end result will meet a predetermined specification.

SQA has proven to be considerably effective within a number of transportation agencies. it is now used to govern a wide variety of construction and material properties, such as pavement strength, thickness, and smoothness. In 1993 a survey reported about half of the states have SQA programs in place and another 25% have statistical specifications in various stages of development

SQA's roots trace back to the early 1960s when the AASHTO's Road Test, a series of accelerated wear tests using various combinations of pavement materials and designs, created an unprecedented awareness of the variability of materials construction methods, and sampling and testing procedures This knowledge has subsequently been shaped into numerous construction specifications based on statistical concepts. Various statistical measures can be used to describe the construction characteristics that are desired and when combined with appropriate acceptance tests, these techniques form the core around which a sound quality assurance program can be based

But engineers were quick to recognize that for many highway items it simply was not possible to define a single level of quality that separated acceptable and unacceptable work. Instead it was necessary to define an acceptable quality level (AQL) that was clearly satisfactory, and a rejectable quality level (RQL) that was so seriously deficient that removal and replacement (or other corrective action) would be warranted. In between these two extremes was a gray area of quality that was too good to require replacement but not good enough to accept for full price. Thus was born the concept of adjusted pay clauses, which provide a way for states to accept marginally defective items for a prearranged level of reduced payment.

Random-sampling plans, statistical acceptance procedures, and adjusted pay schedules were developed for a wide variety of highway items, including bituminous concrete pavement and both pavement and structures made with Portland cement concrete. Initially, SQA specifications used pay schedules that either awarded 100% payment for satisfactory quality or incrementally reduced pay for unsatisfactory work. More recently, many agencies have sweetened the pot for contractors using positive incentives that provide bonuses for quality that substantially exceeds the level defined as acceptable. For example New Jersey may pay up to 102% of the contract value for extremely high quality work.

Weed [1993] in his article titled "the Proof is in the Pavement" explains how New Jersey DOT's Portland cement concrete specification provides an example of most of the basic SQA fundamentals. Although both slump and air entertainment are routinely tested and used to accept or reject the material at the job site, final acceptance is based on 28-day compressive strength. The percent defective (the percentage of concrete estimated to have strength below the specification limit) was chosen as the acceptance parameter because it is believed to be closely related to performance. Historical data were reviewed to determine the levels of percent defective that have been associated with both satisfactory and unsatisfactory performances in the past. The state decided to slightly raise historical quality levels, and new specification limits were set accordingly. Any performance quality related data that could be found or estimated were used along with engineering economics principles to develop a pay schedule that is fair to the construction industry and at the same time protects the interest of the agency.

The development of the concrete specification produced several new ideas, such as the bonus provisions, which were a distinct departure from earlier New Jersey DOT practices. Computer simulation tests, in particular, enabled management to see exactly what the capabilities of the specification were and the probable consequences of various provisions that were under consideration. This put management in a much stronger position to provide the leadership necessary to overcome initial skepticism about SQA.

Several other steps helped New Jersey launch its SQA program. Management had to clearly communicate what it expected of those ultimately responsible for enforcing the requirements of the specification. Training sessions were held to explain how and why statistical specifications work and what they are expected to accomplish. Construction industry representatives were frequently invited to these sessions and were asked for their input. Before the specification was implemented, several field trials were conducted to familiarize everyone with the new procedures. Comments and suggestions were solicited, both from within New Jersey DOT and from industry. This seemed to foster a greater spirit of cooperation among all concerned. The specification was also introduced gradually, with the state cutting all pay adjustments by half for the first two projects. This approach contributed to a generally smooth transition once the concrete specification was finally adopted.

The first pilot project with this specification required 10,000 cubic yards of concrete, including both pavement and structures with an in-place value of about $3 million. The successful bidder subcontracted the concrete production to an independent transit mix producer. Several different mix designs were prepared and tested prior to the start of the job to demonstrate the producer's capability to meet the specification requirements. The compressive strength averaged approximately 1,000 psi higher than would have been necessary to receive 100% payment. As a result the contractor earned bonus payments totaling about $30,000 [Weed, 1993].

The above example clearly demonstrates the need for a new look at the SQA approach. In this overview of the SQA, sampling and testing cost were never mentioned as if it was of no consequence and no direct relation was established between the received product and the potential increase in performance. That is to say in the case of the example given above, where the average strength was determined to be 1000 psi higher than specified, it is not known what incremental impact this would have on the durability or overall performance of concrete, in other words, is it needed. This aspect of the acceptance plan will be challenged in later chapters in this dissertation.

The other and most recent philosophy still in the research phase is the development of Performance-Related Specifications (PRS). This is a long-term multi-million dollars research project sponsored by FHWA. ERES [1993a, 1993b]. Its objectives and basic philosophy are summarized here. Results of the proposed research would enrich a performance based type of specifications.

Through the use of PRS, a methodology is provided through which the quality of pavement construction can be related to the performance and cost of a given project. This ability can lead to the determination of pay adjustments for a specific project. PRS goes beyond the previously mentioned type of specifications by having the ability to identify expected levels of performance associated with specifying different values for a given quality characteristic, whereas most current specifications are unable to identify the expected level of performance. For example, in a PRS for a specified quality characteristic (say strength), the performance of the pavement can be estimated using established relationships, and the level of quality determined in terms of desirable performance (acceptable quality) or undesirable performance (unacceptable quality). In this way, PRS permits the identification and specification of the optimum level of quality that provides the best balance between cost and performance for a given project. It has been estimated use of performance relationships in specifications could produce long-term benefits to users and agencies measured in billions of dollars. The approach taken in this research which is based on Taguchi's methods, to be discussed later, complements in many aspects the research into Performance Related Specifications.

2.5 Pay Adjustments

The FHWA and the state highway agencies, in recent years, have expended a great deal of energy and money toward improving the quality of the nation's transportation infrastructure. Literature reveals a continuous effort towards improving materials and understanding their behavior. New techniques for measuring quality, testing of materials, and incentives and disincentives for contractors are also reported.

In spite of all these efforts, products occasionally fail to comply with specifications It is reasonable to expect that a product, which falls outside specifications will also have reduced performance Given the notion that a particular product falling in this category may not warrant rejection, a common practice in statistically based specification is to adjust the payment for such a product to reflect the level of quality received

In this section, the researcher presents some of the common methods of pay adjustment in use today Later in the research, the philosophy as well as the implementation techniques governing these methods will be evaluated

2.5.1 Life-Cycle-Cost Bases for Pay Adjustments

It has been noted that a major concern for transportation agencies in the development of adjusted pay schedules is the determination of appropriate pay levels for various levels of quality Over the years, several methods have been proposed, and when there was little or no information relating quality measures to performance, the methods have necessarily been quite arbitrary In cases for which quality and performance relationships have been established (or can be estimated), one of the more rational methods for developing pay schedules is based on life cycle costs and the legal principle of liquidated damages In this approach the pay schedule is designed to withhold sufficient payment at the time of construction to cover the cost of future repairs made necessary by defective work.

In the case of highway pavement, for example the thickness and material characteristics are chosen to carry the estimated loading for the desired service life At the end of its useful service life, the pavement will commence receiving a series of overlays that typically last about 10 years each. If the pavement is not capable of carrying the design loading because of construction deficiencies, it will fail prematurely. When this happens, the series of overlays that follow the initial design period will be moved earlier in time, resulting in an extra expense to the transportation agency. Using lifecycle and cost principles, it is possible to develop an expression giving the appropriate pay factor for various levels of expected life.

2.5.2 Pay Adjustment Based on Load Ratio

This method consists of defining discrete intervals for the ratio of expected life to design life (or its equivalent, the ratio of expected load-bearing capacity to the design loading), and then computing the pay factor associated with the midpoint of each interval, making it possible to construct a stepped pay schedule. However, when the operating characteristic curve for a pay schedule such as this is checked, it will usually be found that the provisions for minimum and maximum pay factors have diverted it from the desired curve. This effect is more pronounced with small sample sizes and, if it is considered large enough to require correction, either the sample size must be increased or some of the pay factors in the pay schedule must be modified [Weed,1982].

2.5.3 Pay Adjustment Based on Percent Defective

In the case of pay adjustment based on percent defective, AASHTO design equations are used to determine the expected life estimated from the load carrying capacity for a series of levels of percent defectives of the quality characteristic in question. Since the percent defective is a function of both the mean and the standard deviation, the sensitivity of the procedure to the quality characteristics has not yet been determined.

2.5.4 Justification for *Bonus* Clauses

The bonus clause, sometimes referred to as an incentive provision, was supported for several years by the FHWA on an experimental basis. It was eventually concluded that, in appropriate circumstances, this approach was not only cost beneficial but decidedly in the public interest. Subsequently, all restrictions were removed and the bonus clause has come into more common use for either early completion or superior quality.

For a quality assurance program, it is the latter application that is of interest. In the case of pavement, well-established fatigue relationships demonstrate that improved quality extends the expected service life. Design and construction forces have expressed the opinion that similar benefits would result from higher-quality construction of structures, either by extending the life or reducing the amount of periodic maintenance or both. A longer service life and reduced maintenance translate directly into cost savings for the transportation agency. The bonus provision provides additional incentive to accomplish this by sharing some of the economic benefit with the contractors who are capable of achieving higher levels of quality.

Proponents of pay adjustments and the bonus clause assert that it provides another benefit that should not be overlooked, which is the positive psychological effect of providing a reward for excellent performance rather than just a penalty for poor performance. But the primary justification for a bonus provision has to do with fairness to the contractor. Acceptance procedures based on percent defective (or its counterpart, percent within limits) require at least a small bonus in order to award an average pay factor of 100 percent when the contractor produces quality precisely at the level that has been defined as acceptable. Due to the inherent variability of any sampling and testing process, some samples will underestimate the true quality while others will overestimate it. Unless the acceptance procedure is designed to allow pay incentives (pay factors greater than 100 percent) and pay reductions to balance out in a natural way, the average pay factor will be biased downward at the AQL, and acceptable work will be unfairly penalized. The failure to award an average pay factor of 100 percent at the AQL, even by only one or two percent, can result in many thousands of dollars of unwarranted pay reductions throughout the course of a construction season.

2.6 Acceptance Sampling Plans

In this section the researcher presents an overview of published works which are principally focused on acceptance sampling plans. It has been noted that generally two philosophies dominate over acceptance sampling methodologies. One is referred to as the conventional method, which employs classical statistical principles, and the other as the Bayesian method, which uses Bayesian statistic principles. Followers of conventional statistics are often referred as classical statisticians, and followers of Bayes statistics as Bayesian statisticians.

An important fact to note here is that acceptance sampling plans, whether Bayesian or conventional, are mostly the result of economic considerations. This is due to the fact that quality control inherently necessitates inspections, and total inspection is generally impractical or much too costly. Therefore, representative sampling, where a portion of the given population is used to represent it totally, is the best alternative.

In conventional sampling plans, risk exposure by the producer and the consumer is the primary concern. The resulting sample size, i.e., number of samples is associated with a probability of acceptance and a probability of rejection. As one would reasonably expect, the more the samples the greater the protection. On the other hand, Bayesian economic plans provide an addition to risk considerations, focusing on the cost of the benefit of more samples with respect to the additional protection provided. A Bayesian approach is by far most suitable when optimum acceptance sampling based on cost is the objective.

2.6.1 Risk Based Plans: Conventional Approach

In both conventional and Bayesian methods, acceptance-sampling plans are generally of two types: attributes plans and variables plans. A detailed description of each is provided in the next chapter.

As stated earlier, the U.S. highway transportation industry, according to the literature, almost exclusively uses acceptance plans derived from conventional statistical quality control principles. Willenbrock, Kopac, and Weed have conducted some noted research in this area, Willenbrock and Kopac [1976a], Willenbrock and Kopac [1976b], Willenbrock and Kopac [1976c]. Richard Weed is a supervising statistical engineer with the New Jersey Department of Transportation. His work in the area of quality assurance, quality management, construction management, statistical quality assurance, acceptance testing, pay schedules, and pay adjustment procedures is extensively published. His works relevant to this dissertation are Weed [1982, 1993, 1996]. Willenbrock and Kopac have jointly published several papers in the area of specification writing, operating characteristic curves, and quality control. Their relevant publications are also cited in this project. Kopac currently leads the PRS research referenced in a previous section.

Abdulrahman Abdullah Al Azzam developed, for his Ph.D dissertation at the University of Florida, statistically based acceptance procedures with price adjustment provisions and realistic job mix formula tolerances for flexible pavement materials. The acceptance procedures assumed unknown variability, thus, Al-Azzam used the tables, derived from the incomplete beta function proposed by Lieberman and Resnikoff [1955], to develop quality indexes. Although, the developed procedures are quite general, particular considerations were made to improve the quality assurance programs in Saudi Arabia [Al Azzam, 1993]. It should be noted that Al Azzam research is fundamentally different from this research because the procedures are based on risks, not economics.

In most other industries where the conventional method still prevails, work published by Dodge- Romig [1929], Craig [1941], Blackwell [1947], Wallis[1947], Bowker and Goode[1952], Barnard[1954], Lieberman and Resnikoff [1955], Wetherill[1960], Mace[1964], and Owen[1967] are cited.

2.6.2 Economically Based Plans – Bayesian Approach

Wall and Elsheannawy [1989] in their paper titled "Economically-based Acceptance Sampling Plans" reported that

> Risk based plans are easier to use, but offer less-than optimal in economic terms On the other hand, purely economic plans can be mathematically difficult to design, and are sensitive to estimates the cost involved in acceptance or rejection of a lot Furthermore the task of tracking and estimating the costs can be expensive in itself
> Risks based sampling plans have long been the accepted standards in the practice of acceptance sampling for quality control Usually the economic consequences of a given sampling plan are assessed by experience and intuition at best. With the availability of high speed computers and well-proven statistical methods, economically optimal sampling plans can be developed so that much of the guesswork is removed p 346

When a quality engineer is faced with the task of designing a sampling plan he or she must satisfy the consumer s need for product of a particular quality level within the constraints of the resources available to achieve that quality level Risk-based plans are based on one of two types of risk, producer's risk and consumer s risk The types of risk are discussed latter in chapter 3

While risk based plans are still more commonly used in industry economically based plans offer a practical appeal with their inherent cost advantage Wall and Elsheannawy give two excerpts on the subject, one from Keats (1983) dissertation who so ably points out the needs for economically based acceptance sampling in the following manner

> Users of risk-based plans must decide upon probabilities or assumed risks of accepting bad lots or having good lots rejected The choice of these probabilities or risks is often the result of a mental assessment of the economic consequences of the undesirable results associated with accepting bad lots or having good lots rejected Typically this mental assessment is not sophisticated and the measures of good and bad lot quality are standard values which are established sometime in the past. p 346

The other is from Collins' (1974) dissertation who defines the Bayesian decision theory approach to cost based acceptance sampling as follows:

> The expected cost for a specific sampling plan is obtained by weighting the cost associated with implementation of each lot disposition by the probability that a specific number of defectives is observed in the sample. The total expected cost is obtained by summing the expected costs thus obtained for each possible value which the number of defective may assume. p 346

Dayananda and Evans, Tagaras, Schmidt, Case, and Bennett, Moskowitz and Tang have independently developed models for acceptance plans that aim toward determining the optimum sample sizes implicitly based on cost. Dayananda and Evans [1973] in their publication titled 'Bayesian Acceptance-Sampling Schemes for Two-Sided Test of the Mean of a Normal Distribution of Known Variance' arguably provided the most thorough work in the area of acceptance sampling by variables using several utility functions. The model focuses on developing a procedure for deciding whether a population parameter θ lies in an interval (θ^-, θ^+) where θ^- and θ^+ are specified critical values. In hypothesis terminology the problem is to test the composite Hypothesis H_1 $\theta \in (\theta^-, \theta^+)$ against the composite alternative Hypothesis H_2 $\theta \in (\theta^-, \theta^+)$. Dayananda and Evans also described a procedure for obtaining the optimal decision rule given a sample of observations and for obtaining the optimal sample size when the sampling cost is a linear function of the sample size. Hald [1966], in his single sampling plan economic model, uses a general loss function and a differentiable prior distribution. The model provides good insight towards a better understanding of Bayesian economic modeling. Lehman [1959] gives a comprehensive account of the classical approach to the problem.

Bayesian sampling inspection schemes in which proportion of defectives in a batch have been considered by authors Guthrie and Johns [1959], and Hald [1968] Bayesian solutions to the one sided problem when the mean, θ, is the mean of a normal distribution of known variance have been considered by Grundy Healy and Rees [1956] Raifa and Schlaifer [1961], Schlaifer [1959], Wetherill and Campling [1966] Like Dayananda and Evans Moskowitz and Kwei Tang [1992] provide a Bayesian model using both a quadratic utility function and a step utility function Sensitivity analysis for both cost functions show that the optimal sampling plan is robust with respect to the form of the prior distribution, as well as to misspecification of its mean and variance, so long that the tail specification reasonably approximates that of a normal distribution

Schmidt, Case and Bennett [1974] presented two economic models using no loss function directly; one of the models assumes that lots not meeting specification are screened, so it includes a screened cost in the formulation. These two models are well suited for agencies that prefer the goal post type of specifications, where no quality loss is assumed to occur within the range of specifications. The Three Action Cost Model for Acceptance by Variables published also by Schmidt, Case, and Bennett [1980] is very similar to the models referenced earlier with only minor modifications. The researcher found their analysis of cost components very insightful George Tagaras [1994] in his paper titled Economic Acceptance Sampling by Variables with Quadratic Quality Costs' followed Schmidt, Case, and Bennett [1980], and Moskowitz and Kwei Tang approach to the problem. Tagaras model is closely followed in this research. Ikuo Arizono et al [1997] developed a model that is indexed by Taguchi quadratic loss function. It is the only model, which explicitly addresses the risks involve in sampling. However, the researcher finds such a model limited because of its narrow cost structure

Several other authors such as Montgomery [1995] Ronen [1995] Breakwell [1956], and Moura [1991] have also made significant contributions

The literature search reveals no economically based plans in use by the U S highway industry

2.7 The Taguchi Philosophy

Roy[1990] gives an interesting summary of the Taguchi philosophy The researcher reinforced this literature with the work of Ross[1996] Since the late 1940s, Dr Genichi Taguchi has introduced several new statistical concepts, which have proven to be valuable tools in the area of quality improvement. A significant number of Japanese manufacturers have used his approach for improving product and process quality with unprecedented success The widespread application of the Taguchi Method is largely responsible for the high quality of Japanese automobiles.

Taguchi's method is based upon an approach, which is completely different from the conventional practices of quality engineering The methodology emphasizes designing the quality into the products and processes, whereas the more usual practice relies upon inspection. In his quality improvement practices Taguchi essentially utilizes the conventional statistical tools but he simplifies them by identifying a set of rigid guidelines for experiment layout and the analysis of results Taguchi's approach has been extremely effective in improving the quality of Japanese products. Recently Western industries have begun to recognize Taguchi's method as a simple but highly effective approach to improving product and process quality [Roy,1990]

Taguchi put forth an excellent philosophy for quality control in the manufacturing industries. Indeed, his doctrine is creating an entirely different breed of engineers who are highly concern about quality issues. He has in fact fathered a new quality culture in this country. Ford Motor Company, for example, requires that all Ford Motor and suppliers engineers are trained in Taguchi methodology and that these principles be used to resolve their quality issues. Although founded on three very simple and fundamental concepts, his philosophy has far reaching consequences. These concepts are summarized below

(1) Quality should be designed into the product and not inspected into it.

(2) Quality is best achieved by minimizing the deviation from a target. The product should be so designed that it is immune to uncontrollable environmental factors.

(3) The cost of quality should be measured as a function of deviation from the standard and the quality losses should be measured system-wide.

Taguchi used as foundation W. E. Deming's observation that 85% of poor quality work is attributable to the manufacturing process and only 15% to the worker. Hence he developed manufacturing systems that were "robust" or insensitive to daily and seasonal variations of environment, machine wear, and other external factors. The three principles were his guides in developing these systems, testing the factors affecting quality production, and specifying product parameters.

Taguchi believed that the best way to improve quality was to design and build it into the product. Quality improvement starts at the very beginning, i.e., during the design stages of a product or a process, and continues through the production phase. He proposed an "off-line" strategy for developing quality improvement in place of an attempt to inspect quality into a product on the production line. He observed that process inspection, screening, and salvaging couldn't improve poor quality. No amount of inspection can put quality back into the product; it barely treats a symptom. Therefore, quality concepts should be based upon, and developed around the philosophy of prevention. The product design must be so robust that it is immune to the influence of uncontrolled environmental factors on the manufacturing processes. He emphasized that quality is what one designs into a product.

Taguchi's second concept deals with actual methods of effecting quality. He contended that quality is directly related to deviation of a design parameter from the target value, not to conformance or to some fixed specifications. A product may be produced with properties skewed towards one end of an acceptance range yet show shorter life expectancy. However, by specifying a target value for the critical property and developing manufacturing processes to meet the target value with little deviation, the life expectancy may be much improved.

Taguchi's third concept calls for measuring deviations from a given design parameter in terms of the overall life cycle costs of the product. These costs would include the cost of scrap, rework, inspection, returns, warranty service calls, and product replacement. These costs also provide guidance regarding the major parameters to be controlled [Roy 1990].

Taguchi views quality improvement as an ongoing effort. His aim is to continually strive to reduce variation around the target value. A product under investigation may exhibit a distribution that has a mean value different from the target value. The first step towards improving quality is to achieve the population distribution as close to the target value as possible. To accomplish this, Taguchi designs experiments using especially constructed tables known as "orthogonal arrays" (OA). The use of these tables makes the design of experiments very easy and consistent.

A second objective of manufacturing products to conform to an ideal value is to reduce the variation or scatter around the target. To accomplish this objective, Dr Taguchi cleverly incorporates a unique way to treat noise factors. Noise factors, according to his terminology, are factors that influence the response of a process but cannot be economically controlled. The noise factors such as weather conditions, machinery wear etc., are usually the prime sources for variations. Through the use of what he calls the outer arrays, Taguchi devised an effective way to study their influence with the least number of repetitions. The end result is a "robust" design affected minimally by noise, i.e., with a high signal to noise ratio.

To achieve desirable product quality by design, Dr. Taguchi recommends a three-stage process:

- System design
- Parameter design
- Tolerance design

The focus of the system design phase is on determining the suitable working levels of design factors. It includes designing and testing a system based on the engineer's judgment of selected materials, parts, and nominal product/process parameters based on current technology. Most often, it involves innovation and knowledge from the applicable fields of science and technology.

While system design helps to identify the working levels of the design factors, parameter design seeks to determine the factor levels that produce the best performance of the product/process under study. The optimum condition is selected so that the influence of the uncontrolled factors (noise factors) causes minimum variation of system performance.

Tolerance design is a step used to fine tune the results of parameter design by tightening the tolerance of factors with significant influence on the product. Such steps will normally lead to identifying the need for better, newer equipment, spending more money for inspection, etc.

2.7.1 The Concept of the Loss Function

The concept of the "total loss function" employed by Dr. Taguchi has forced engineers and cost accountants to take a serious look at the quality control practices of the past. The concept is simple but effective. He defines quality as "the total loss imparted to the society from the time a product is shipped to the customer." The loss is measured in monetary terms and includes all costs of a perfect product. The definition can be expanded to include the development and manufacturing phases of a product.

A poorly conceived and designed product begins to impart losses to society from the embryonic stage and continues to do so until steps are taken to improve its functional performance. There are two major categories of loss to society with respect to the product quality The first category relates to the losses incurred as a result of harmful effects to society (e g , pollution) and the second relates to the losses arising because of excessive variation in functional performance For the purpose of this research, the total loss function refers essentially to the second category

The conventional method of computing the cost of quality is based on the number of parts rejected and reworked This method of quality evaluation is incapable of distinguishing between two samples, both within the specification limits but with different distributions of targeted properties Figure 2 1 below shows the conventional method and Taguchi s view of loss function This graph depicts the loss function as a function of deviation from an ideal or the target value of a given design parameter.

Here T represents the target value or the most desirable value of the parameter under consideration. This parameter may be a critical dimension, color of the product, surface finish or any other characteristic that contributes to the customer's conception of quality How this ideal value is used in achieving the quality goals will be evident later

UAL and LAL in Figure 2 1 represent upper and lower acceptable limits of a design parameter, respectively Normally the product is functionally acceptable if the value of the specified parameter is within the range between the UAL and LAL. No societal loss is assumed to occur and the product is shipped to the consumer However, outside these limits (as shown by the crosshatched region) 100% functional deterioration occurs and the product is either discarded or subjected to salvage operations. Every attempt is made to control the manufacturing process to maintain the product within these limits

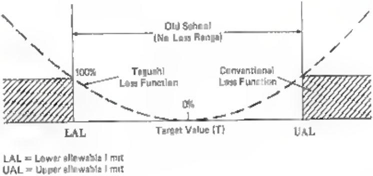

Old School
(No Loss Range)
100%
Taguchi
Loss Function
Conventional
Loss Function
0%
LAL
Target Value (T)
UAL
LAL = Lower allowable limit
UAL = Upper allowable limit

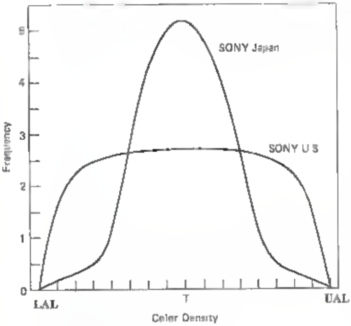
Figure 2 1 Taguchi and conventional loss function

However, according to Taguchi, there is no sharp cutoff in the real world. Performance begins to gradually deteriorate as the design parameter deviates from its optimum value. Therefore he proposed that the loss function be measured by the deviation from the ideal value. This function is continuous as shown by the dotted line in Figure 2.1. Product performance begins to suffer when the design parameters deviate from the ideal or the target value. Taguchi's definition clearly puts more emphasis on customer satisfaction, whereas previously all definitions were related to the producer. Optimum customer satisfaction can be achieved by developing the products that meet the target value on a consistent basis. It may be worthwhile to mention that Taguchi allows for more than 100% loss imparted by a product. Such cases can occur when a subsystem results in a failure of the entire system or when a system fails catastrophically. Thus, the single most important aspect of Taguchi's quality control philosophy is the minimization of variation around the target value [Roy, 1990].

A case study conducted by SONY makes it abundantly clear that these two schools of thought are significantly different from each other, and indeed effect customer satisfaction. SONY manufactures color TVs in Japan and in the U.S. The TVs from both sources are intended for the U.S. market and have identical design and system tolerances. Yet, American consumers consistently preferred the color characteristics of the TV sets manufactured overseas.

A study was conducted to determine a cause for the difference in customer preference. The results indicated that the frequency distributions for the sets manufactured in U.S. and those manufactured in Japan were significantly different as shown in Figure 2.2. Plants in both countries produced TVs with color density within tolerance range. No TVs with out-of tolerance color characteristics were shipped to the consumer. However, the U.S. built sets followed a somewhat flat distribution constant with a go/no go philosophy, while the product manufactured in Japan followed a normal distribution with small deviation from the target value. The large scatter observed in the performance characteristics of the product manufactured in the U.S., as is evident from Figure 2.2, was responsible for significantly lower customer preference for these sets. Once the process in the U.S. plant was brought under control and began to produce the frequency distribution similar to the TVs produced in Japan, customer satisfaction with the U.S. product achieved the level of satisfaction seen with the imported sets. The Sony case demonstrated that quality is more than just producing between upper and lower quality limits, it is achieving the target as much as possible and limiting deviations from the target [Roy 1990].

Consider another example, which will further support this concept of quality. Two batches of main bearings for an internal combustion engine were received from two different sources, A and B, for a new engine development program. Under laboratory conditions, bearings from source B wore much faster than those from source A. To pinpoint the cause of unequal wear selected performance characteristics of the bearings were measured and posted. Both batches of bearings were within the design specifications. However, the source B bearing consistently measured a mean diameter on the larger side of the tolerance limits as depicted in Figure 2.3 resulted in excessive



LAL – Lower allowable limit
UAL – Upper allowable limit
T – Target value

Figure 2.2 TV color density distributions

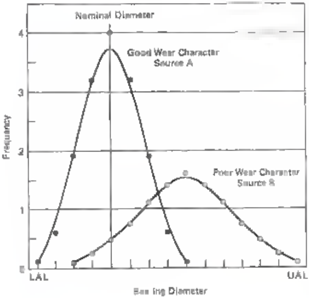


Figure 2.3 Bearing diameter distribution

clearance. Bearing analysis later revealed that excessive clearance adversely effected the oil film thickness, causing the poor wear properties of this batch. Adjusting the manufacturing process to maintain bearing diameter near the target value solved the problem, [Roy,1990]

The loss function and its applications are discussed in detail in later chapters (3 4), but at present, it is important to note that the quality loss function is a continuous function and is a measure of deviation from the target value. The conformance to specification limits, LAL and UAL is an inadequate measure to define the quality loss function.

Quality loss is related to product performance characteristics and can best be minimized by designing quality into the product. Prevention of poor quality is less costly than rework, and yields far better returns. Quality loss results from customer dissatisfaction and should be measured system-wide rather than at a discrete point in the manufacturing process. The research concludes that minimization of quality loss is probably the best to improve and maintain the quality of highway transportation infrastructure.

CHAPTER 3

REVIEW OF BASIC STATISTICAL, MATHEMATICAL, AND ECONOMIC ANALYSIS PRINCIPLES UNDERLYING THE OPTIMUM SAMPLING PLAN MODEL

The purpose of this chapter is to present, briefly, an overview of the statistical, mathematical, and economic concepts necessary to understand the concept of acceptance sampling in general, and in particular, the optimum sampling plan model which constitute the bulk of this research. In the interest of simplicity, rigorous mathematical formulations or proofs of these concepts are avoided wherever possible.

3.1 Statistical Modeling

An appropriate starting point for this chapter is to consider the definition given by Shapiro and Gross [1981]. A statistical model is a mathematical formulation that expresses in terms of probabilities the various output of a system. One finds a statistical model most useful in situations where the output cannot be expressed as a fixed function of the input variables. In those cases, one would need to consider closely the actual operation and the performance of the systems. For example, consider the measurement of thickness of a concrete pavement. Assuming several measurements were taken, one would not be at all surprised to find different reading for each measurement. These measurements can be classified as the output of the system. It can be further assumed that the actual thickness of the pavement is fixed and that these variabilities in the reading are due to errors in measurement. Therefore a model is selected to represent this variability Thus the system output combines the true thickness of the pavement and the measurement error It follows that the model

$$y_i = \mu + \varepsilon_i \tag{3.1}$$

expresses this combination in a simple way, where

y_i = the output i.e. the ith measurement

μ = the true thickness of the pavement

ε_i = measurement of the error for the ith trial

In order for equation 3.1 to be considered a statistical model, it is necessary to select a probability distribution to represent the variability ε_i. Since variability is an inherent property of almost every system in nature, it makes good sense to extend this concept to quality control systems more specifically, for the purpose of this research to acceptance sampling. In addition to inherent product variability many other types and sources of variability associated with acceptance sampling will be discuss later

The model developed above is quite simple, but is has many of the features of more complex examples One should bear in mind that a statistical model such as the one representing the example given above, is an idealization of the system, it is not exact Usually the more complex the system, the less exact the model. This is true because of the difficulty of reducing the output of complex system to mathematical expressions and the inability to manipulate highly complex expressions A common dilemma is that the more the model is simplified, the easier it is to analyze but the less precise the result

It is fitting to point out that, statistics as a field of mathematics, provides many tools to deal with complex situations. The idea that the measure of a characteristic must be unbiased is a very good example. This simply means that the average measurement error must be zero. This notion further implies that the ε_i are sometime positive and sometime negative, but taken over a large number of observations ε_i will average zero so the net result is simply μ. One must keep in mind that, in a real problem there is only a limited number of data points; and because of this fact, only an approximation of μ is obtained. Thus, an estimate of the variability of the measuring error is required. In statistics, this estimate of variability is called variance and is represented by the symbol σ^2. Later in this project the formula for determining the variance and other related parameters will be presented and discussed.

The researcher wants to direct special attention to the importance of the basic model, just presented in this section of this dissertation, which throughout this dissertation is used as the key element in the foundation upon which other models are developed.

3.2 The Concepts of Quality and Variability

In acceptance sampling inspection, as well as other types of sampling inspection one is usually concerned with two aspects of quality. The first one has to do with the determining the quality of an individual unit, and the second one has to do with determining the over-all quality of the lot.

In general, there are several types of inspection that can be used to evaluate and describe the quality of a unit of product. The choice of a method usually depends on the characteristic being measured and on the device most appropriate for the job. Among the many such methods of measuring quality, only *attributes* and *variables* are discussed here due to their relevance to this research.

When it comes to measuring the quality of a lot, on the other hand, other methods are used. Again for the purpose of this dissertation and the nature of the product under consideration here, only the *percent defective* (or percent within limit), the *standard deviation of item qualities in the lot* and the *lot by-lot* methods are considered.

Willenbrock [1976], in A Manual for Statistical Quality Control of Highway Construction, considers the many sources or components of variability. These components are briefly discussed in this section. Typically when the standard deviation of a population is measured, it is the overall standard deviation, σ, that is obtained. It's the sum of the entire individual internal standard deviations discussed below. The Overall standard deviation is the primary value that should ultimately be related to the specification limit, within the timeframe defined by the lot. The lot is defined as the time, area, volume, length, etc. over which the product or construction will be judged for acceptance or over which the quality control limits are established. The individual sources of variability should never be used by themselves to determine the specification limit.

In his manual, Willenbrock provides an explanation of the Components of variability which are summarized here. There are five major components of variability of product characteristics that should be discussed. An understanding of the relationship and interaction between them will indicate how the overall variability (i.e., the variability that must be considered when specifications for a particular product are being developed) may be determined. The types of variability discussed are

- Inherent
- Sampling and Testing
- Within Batch
- Batch-to-Batch
- Overall

It should be noted that in the development of a specification, each component should be examined with respect to a number of other methods of analysis also good judgement should be used in order to insure that the individual components do not bias the overall variability and ultimately the specifications. If the components are excessively variable, every effort should be made to reduce the variability. If the variability of the components is small, they may cause the specification to be too restrictive. In any case, these components should be identified and quantified in order to determine the source and magnitude of the variability.

The inherent variability is the true random variability of the product. It is a function of the characteristics of the product itself. It may vary in magnitude, but it is surprisingly one of the smallest sources of variability. This source of variability cannot be used by itself as the specification limit. Inherent variability, like other sources of variability, can only be determined by the process of sampling and testing. It should be recognized that this sampling and testing would introduce additional sources of variability.

The sampling and testing components are actually separate sources of variability. The are often combined into one source of variability. Sampling variability is a function of sampling technique and is detected when a sample increment taken from one part of a batch will not indicate the same test result as one taken from another part of the same batch. Testing variability is the lack of repeatability of test results between test portions that may include the effects of reducing sample increments to test portion size. Operators, equipment condition, calibration, and test procedures are a few of the important factors that can cause high testing variability. Sampling and testing variability are sometimes difficult to separate from other sources of variability because samples have to be taken and tests have to be made on a product to determine the variability. As a result, they become an integral part of the overall variability.

Within-batch variability depends upon the magnitude of the difference in the measurements between two increments that are taken from the same batch. It should be noted that the same thing could be said about sampling variability. Sampling variability is the function of sampling technique, while within-batch variability is a function of the consistency of the process. Classic examples of within-batch variability are aggregate segregation, slump change from the front of the load to the back, and variability in core depths of a concrete pavement for adjacent cores in the same location.

Batch-to-batch variability is usually the largest source of variability in any process. It represents the difference in test results from one batch to other batches of the same product from the same process. It is always caused by the process and is greatest when the process is termed "out-of-control." To detect this type of variability it is necessary to expose the process to sampling for a long period of time.

The sum of all of the individual sources of variability is called the overall variability. It becomes the primary consideration when establishing specification limits. It is very important to remember that, in order to properly determine specification limits, all of the sources of variability must be included, even though the overall variability is the primary consideration for the selection of these limits

3.3 Fundamental Concepts in Statistical Acceptance Sampling

In order to have a better understanding of the mechanisms of acceptance sampling one need first understand the purpose of acceptance sampling. The following summary is aimed at satisfying this need. When an agency purchases raw materials, or finished products, the purchase contract will often indicate that the acceptance of the products will depend upon compliance with certain quality specifications.

Both the purchasing agency and the supplier understand that, if the products are made by mass-production methods (or in the case of pavement, concrete to be placed in a day's work is mixed in batches), some will not meet the specifications. Furthermore, the number and location of these defectives in any lot, will rarely be known. Bowker and Goode [1952] have concluded that some form of acceptance sampling is desirable for the following reasons. first, to protect the purchasing agency against the acceptance of an undue quantity of defective items; second, to exert pressure on the supplier to improve his product when necessary; and third, to assist quality control in the reduction of production costs when the product is transferred within an organization. To implement this concept, a method of sampling must be devised. Since a screening method of sampling, where elimination of defectives is essential, is not practical in pavement. Under the latter method, the entire submitted concrete batches would have to be individually inspected and individually accepted or rejected. The cost of inspecting each truck may be too high, or the necessary tests may cause undue delay to the contractor. In addition, screening cannot guarantee the removal of all defective items. It is impossible for inspectors to work with 100 percent accuracy, especially when a large number of items are inspected. In some cases, sampling inspection with its lower costs may give the same quality assurance.

A method often used by transportation agencies is the control charts method but this is usually done in addition to some other form of acceptance sampling method. The usefulness of control charts is found in their ability to supply useful information about the quality level of the product and about the degree of control at the various production processes used in its manufacture. The method of acceptance sampling selected by the researcher in this dissertation is the single sampling lot-by-lot technique. It has been found to be the most satisfactory acceptance sampling method because where sound statistical plans are used it can provide any desired degree of protection.

Acceptance sampling has the advantage of exerting more effective pressure for quality improvement than is possible with 100 percent inspection. As stated earlier, inspection, in the sense of sorting good products from bad ones cannot be relied on to ensure that all accepted products are good. Obviously, no sampling procedure can eliminate all defectives. It follows that the best way to be sure that accepted product is good is to have the product made right in the first place. Deming [1950] asserts that the rejection of entire lots brings much stronger pressure for quality improvement than the rejection of an individual item.

3.3.1 Acceptance Using Lot by Lot Method

In lot-by-lot acceptance, one considers each submitted lot of product separately and bases the decision to accept or reject the lot on the evidence of one or more samples chosen at random from the lot. This type of plan, as the one proposed in this dissertation is described as a single sampling plan.

When sampling plans are properly constructed, a large proportion of the better quality lots will be accepted and a large proportion of the low quality lot will be rejected. In the highway construction industry, rejected lots are sometimes "screened" and resubmitted for acceptance. Here screen simply means that additional samples are taken and the lot is re-evaluated (for Portland cement concrete compressive strength only). It is not used to indicate complete inspection of un sampled products. In the accepted lots the average percent defective will be less than the average percent defective in all the lots originally submitted.

Bowker and Goode [1952] explain that lot-by lot sampling inspection improves quality in several ways. Inspection by lots lowers the number of defective items per accepted lot as compared with the number of defectives in the lots taken as a whole. Because a large of number of rejected lots is costly to the supplier, the supplier will try hard to submit better quality lots in the future. In the highway industry, lot-by-lot method can have a psychological advantage over the skip lot method, which would be less expensive.

3.3.2 Concept of Sampling by Attributes

Bowker and Goode [1952] define sampling by *attributes* as go and no-go gauge systems for checking dimensional properties of parts. This type of quality measuring plan is appropriate in dichotomous situations where inspection results can only be classified into two classes of outcomes. The *attributes* single sampling plan is applicable to all sampling situations because of its simplicity, and it has become the benchmark against which other plans are judged.

Dodge and Romig [1929] developed The Military Standard 105D, which is the most popular reference for attributes sampling. Although sampling inspection by attributes remains the most commonly used method to measure quality, it has limitations.

3.3.3 Concept of Sampling by Variables

A very brief discussion on sampling by attributes was presented in the section above. While an important feature in acceptance inspection, it is not the focus of this dissertation, sampling by variables is the focus.

It was noted previously that attributes acceptance sampling plans are the most utilized in industry in general and will without any doubt continue to be so the future. This generalization does not include the highway industry where variables sampling plans are preferred. Yet, there has been a considerable increase in the use of acceptance sampling by variables in main stream industry; and it appears that this tendency will likely continue in the future.

When a quality characteristic is such that is can be measured on a continuous scale and its underlying distribution is normal, sampling by variables should be the method of choice, even though almost all quality characteristics can also be evaluated using attributes criteria. Unfortunately the same cannot be said in the case of variables Sampling by attributes is qualitative where as sampling by variables is quantitative.

Grant [1964] Schilling [1982] and Bowker and Goode [1952] have independently concluded that for those quality characteristics that can be measured, the cost of inspection per item is less by attributes than by variables. This is due to the greater economy of inspection methods using the go/no-go principle The fact that acceptance criteria must be applied separately to each quality characteristic is perhaps the most serious limitation on the use of sampling by variables This tends to increase the cost of acceptance inspection

In spite of the limitations cited, acceptance sampling by variables is often preferable to acceptance sampling by attributes, particularly for those quality characteristics that are the source of troubles, i.e., compressive strength of concrete, because acceptance sampling by variables provides more information about the quality characteristics under consideration.

Acceptance sampling by variables may lead to a number of desirable results, as follows

1. For the same sample size, better quality protection can be obtained with variables criteria than with attributes Stated a little differently, for a given quality protection against various possible percentages of defectives smaller samples can be used with variables than with attributes
2. The extent of compliance or noncompliance to the desired value of a quality characteristic is given weight where variables criteria are used. This can be important wherever there is a margin of safety in the design specifications or e

gray zone of values of the quality characteristic between clearly satisfactory and clearly unsatisfactory

3. Variables information usually gives a better basis for guidance toward quality improvement.
4. Variables information provide a better basis for giving weight to quality history in acceptance decisions
5. Errors of measurement are more likely to be revealed with variables information

3.3.4 The Assumption of a Normal Distribution

In the earlier discussion of the limitations of sampling inspection by variables, it was stated that values for the quality characteristic being inspected must be normally distributed, or almost normally distributed. That is to say, if all the items in the lot were to be measured, the measurements would be distributed among the possible values in a manner similar to the distribution of compressive strength of concrete data indicated by the curve represented in Fig. 3.1

The most obvious features of the normal distribution are symmetrical distribution of readings on each side of the average and the clustering of readings about the average or the *mean*. Ostle and Malone [1988] pointed out that this grouping of data around the *mean* follows a theorem known as the central limit theorem. Ostle further comments that this theorem is without a doubt the most important one in all of statistics. The central limit theorem states that *if a population has a finite variance* σ^2 *and a mean* μ, *the*

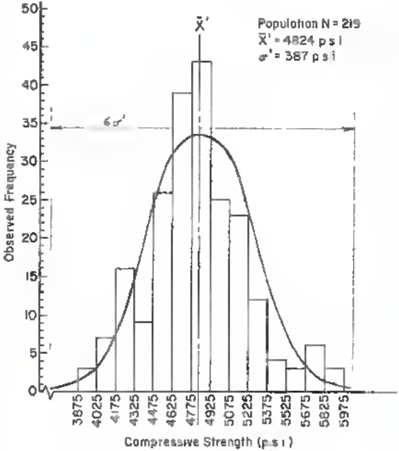


Figure 3 1 Normal distribution of Portland cement concrete strength data

distribution of the sample mean approaches the normal distribution with variance σ^2/n and mean μ, as the sample size increases The pattern of the frequency rates, as values deviate from the average should be noted in particular Because for values near the average the rate of decrease is slight. Then for a considerable range of values the rate of decrease is sharp, until the frequencies become relatively small when it slackens and approaches zero This pattern of decreasing frequencies gives the so-called bell shape to the normal distribution. The variance σ^2 and its square root, the standard deviation, are measures of scatter or dispersion of measurements around the *mean*.

In Juran's Handbook for Quality Control [1988] one finds the equation that can be used to calculate the shape of this curve.

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X_i - \mu)^2}{2\sigma^2}} \tag{3.2}$$

In order to obtain quality protection close to that expected from the variables plans, the distribution of values within the inspection lot must approximate the normal distribution All calculations used in constructing acceptance plans are based upon the normal distribution assumption Thus, one reason why the central limit theorem is so important is that no matter how the population is distributed, provided it has a finite variance the sample mean will be approximately normally distributed if and only if the sample size is sufficiently large.

The density functions for the sample mean for several sample sizes are presented in Figure 3 2. Notice that the initial population is not normal, but as the sample size increases, the picture of normal distribution is clearly evident. Figure 3 3 shows additional illustrations of the impact of the central limit theorem on the sampling distribution of the mean It is also important to note that normality begin to take shape at n=5 this is one of the reasons why a minimum sample of size 5 is generally used

It is true that properties of most construction raw materials used by the highway industry and of most manufactured products seem to be distributed in forms that are close enough to the normal distribution for the practical use of these variables plans The main concern of transportation agencies should be to acquire the ability to recognize the occasional exceptional cases, so that appropriate allowances can be made or other methods of acceptance inspection used. If for some other reason a normal distribution is not approximated, one alternative to variable acceptance sampling is the combined use of variables and attribute plans, another alternative is to make use of the non central t distribution when applicable.

3.3 5 Skewed Distributions

The most common exception to normality is the skewed distribution. In fact, it seems reasonable to assume that a small amount of skewness is always present The frequencies in a skewed distribution are not distributed symmetrically about the mean but take the form shown by Fig 3 4 The most frequent value, the mode lies to one side of the mean.

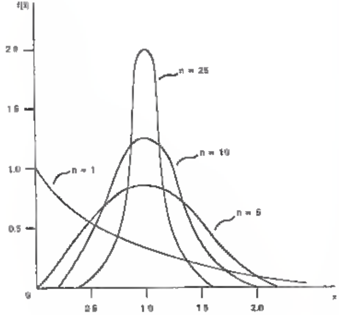


Figure 3.2 Density function for the mean of an exponential population for various values of "n"

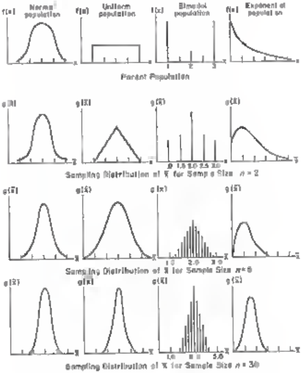


Figure 3 3 Sampling distribution of the mean for various population distributions when the sample size, n=2, 5, 30

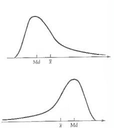


Figure 3.4 Graphs of positively and negatively skewed distributions

3.3.6 Notes on Acceptance Criteria

It was stated earlier that the most important consideration in applying variables sampling plans is the requirement that the shape of the underlying distribution of measurements to which the plan is to be applied must be known and stable. This means that probability plots or statistical tests on past data must show that the distribution of measurements involved actually is that assumed by the plan. For a normal distribution, it is the underlying distribution of measurements, which relates the proportion of units outside the specification limit to a fixed position of the population mean of the measurements. Then variables plans for process parameter can be used to confirm or deny that the population mean is in the proper position. In a nutshell, this is how variables sampling works. It must be emphasized that the underlying distribution must be known to be that assumed by the plan for variables sampling to be properly applied.

The basic theoretical nature of variables acceptance sampling plans is illustrated in Fig. 3.5, which involves an upper specification limit and assumes the underlying distribution of individual measurements to be normal. If the procedure of Fig. 3.5 is applied, the mean $\overline{X}$ of a sample of 'n' measurements is compared to an acceptance limit A and the lot accepted if $\overline{X}$ is not greater than A. Figure 3.5 shows that if the distribution of individual measurements is as shown, with σ known, a proportion p of the product above the upper specification limit U implies the mean of the distribution must be fixed at the position indicated by, μ. Sample means of size 'n' then would be distributed about, μ as shown and so the probability of obtaining an $\overline{X}$ not greater than A is as indicated by the shaded area in the diagram [Schilling 1982].

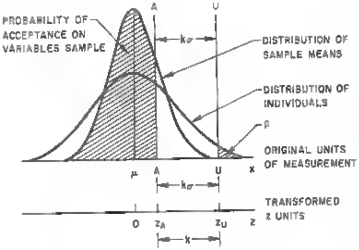

A
U
PROBABILITY OF ACCEPTANCE ON VARIABLES SAMPLE
kσ
DISTRIBUTION OF SAMPLE MEANS
DISTRIBUTION OF INDIVIDUALS
p
ORIGINAL UNITS OF MEASUREMENT
μ
A
U
x
kσ
TRANSFORMED z UNITS
0
z_A
z_U
z
k

Figure 3.5 Distributions in variables sampling

If the shape of the underlying distribution of individual measurements were known, acceptance sampling could be performed directly on the measurements themselves. Such procedures form the basis for variables sampling plans for percent defective and, when applicable, provide considerable savings in sample size. The basic idea of variables sampling for percent defective is to show that the sample results are sufficiently far within the specification limit(s) to assure the acceptability of the lot with a reasonable probability.

In highway construction, there are three types of variables sampling plans that are generally used. A brief explanation of each is given here. Consider first the average value plan. Suppose that it is of interest to evaluate the compressive strength of concrete for compliance with specifications. The highway agency might want the arithmetic mean of compressive strength of concrete to be $\mu = 5000$ psi and the standard deviation, σ to be 300 psi. Following the common practice of the highway industry, the acceptance sampling plan might state that a sample size $n=5$ sublots is to be taken at random from a lot, and that the average compressive strength $\overline{X}$ for the sample must be within the following tolerance range in order to be acceptable

$$4730 \le \overline{X} \le 5270$$

and $\overline{X}$ is calculated from the equation.

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (3.3)$$

The values obtained from this example are commonly referred to as the lower tolerance limit (L.T.L.) and upper tolerance limit (U.T.L.), respectively. They can also be considered as acceptable quality levels for compressive strength of concrete. So any sample average that is less than these values is cause for rejecting the lot. On the other hand any average value between them and the lot is accepted. It should be kept in mind that in using this approach, one is making a statistical inference about the population based on the average compressive strength of concrete of the sample in accordance with the indicated acceptance plan.

The second method is actually an extension of the first one describe previously. It is referred to as average value and standard deviation plan s. This type of plans includes both an evaluation of the sample mean $\bar{X}$ and the standard deviation s. An acceptance decision for this plan is made by taking a sample at random from a lot, and the sample statistics $\bar{X}$ and s are then determined. The specification limits define a passing test result, and like in the first case, are referred to as the L.T.L. and U.T.L. values. A passing $\bar{X}$ would fall within the following limits

$$LTL \leq \bar{X} \leq UTL$$

It is also required that a percent defective goal defining a quantity equivalent to the acceptance number c in attribute plans. An important point to be made is that this equivalence to the attributes sampling is done to allow the comparison between attributes and variables plan using the same operating characteristic curve. This number is designated by the letter k and the rule is that material is acceptable if

$$\bar{X} + ks \leq UTL$$

and

$$\bar{X} - ks \leq LTL$$

The standard deviation s is calculated from the equation

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x - \bar{x})^2}{n - 1}} \tag{3.4}$$

In addition to acceptance constraints given above, it is required that s ≤ maximum allowable standard deviation value.

The third method of acceptance sampling by variables that will be discussed in this dissertation is the percent within limits or its counterpart, percent defective. This plan is actually very similar to the second plan just defined. The difference is that, in lieu of stating that the average or standard deviation of sample of size "n" is to be within some specified limits, it is more concerned with answering the question "what percent of the material in the lot is within the specification limits (or what percent of the material in the lot is outside the specification limits)?"

Because in the last two methods the variability is being estimated by the standard deviation computed from the sample, acceptance-sampling plans of this type are referred to as "variability unknown" plans.

In order to estimate the lot percent defective or percent within limits, it is first necessary to calculate one or two quality index values (Q). One value is used if the situation calls for a single specification limit and two values if the situation calls for a double specification limit The appropriate equations to compute this are given below

$$Q_L = \frac{\bar{x} - L}{s} \qquad (3\ 5)$$

and

$$Q_U = \frac{U - x}{s} \qquad (3\ 6)$$

where

Q = quality index

$\bar{x}$ = sample mean

s = sample standard deviation

L = lower specification limit

U = upper specification limit

Recall the earlier definition given for the AQL value, in the interest of completeness it can be formally restated as follows according to AASHTO R9-R90 AQL is that level of percent defective at which the work is considered completely acceptable There is also a RQL value that is defined as the level of percent defective at which the work is considered unacceptable and repair or replacement is warranted

It should be noted that it is common practice to established the AQL and RQL values prior to the development of the plan and in this process must not take in consideration any risk

3.4 Lot Size

The definition of lot size, or what constitute a lot is very important. If there is one significant departure from the existing specifications being used it occurs at the point where a lot is defined. We can define a lot as a prescribed and defined quantity of material (whether it is by area, tonnage, units, etc.) that is manufactured from the same process for the same purpose. The contractor for quality control purposes may define a lot size. This lot may or may not be the quantity that is offered for acceptance as a unit. All sampling, testing requirements, and considerations are defined for that quantity. Only by establishing the size of the lot can one select the proper sampling locations and frequencies for either quality control purposes or to estimate the quantity of the material that is within the specified limits.

If one considers the concept of lot-by-lot testing for acceptance purposes for example, the process of constructing a highway may be thought of as the production of a succession of lots. These lots are sequentially presented to the engineer for acceptance or rejection. In order to implement the acceptance plan, each lot is considered to be made up of subdivisions of equal sizes called sublots. Sampling locations are randomized within the boundaries of each sublot. The principal advantage of this concept is to stratify or separate the lot into smaller components. This is done in order to eliminate the possibility of the random samples being clustered around the same location. The samples are to be distributed throughout the lot. When estimating the size of lots and sublots, the subject of risks arises. When the lot size is too small, the acceptance plan becomes burdened with an excessive amount of costly testing. More importantly the contractor reduces his risk considerably since smaller parcels are submitted for acceptance at any one time. This essentially minimizes his economic liability to keep his process in control. However, too large a lot size puts the contractor at a distinct disadvantage because of his increased risk of having a very large quantity of material rejected (or penalized).

3.5 Acceptance Plans Decision Errors (α,β)

Whenever a decision about the acceptability of a lot of material is based on the result of a small sample of size n which is obtained from that lot, errors in judgment are possible. In other words, no acceptance procedure based upon sampling can assure the acceptance of only good items. In fact, according to Juran and Gryna [1988] and Bowker and Goode [1952], acceptance sampling plans occasionally will permit the acceptance of a lot with a relatively high percentage of defective item and the rejection of a lot containing relatively low percentage of defectives. The element of chance cannot be avoided in the drawing of sample items from a lot. Consequently, the quality of items in the sample may occasionally depart considerably from the quality of the items in the lot.

In the case of pavement acceptance sampling, the transportation agency in making a decision about the acceptance of a lot has only two choices. it can either accept or reject the lot based on the sample evidence. It should be noted that these two choices have four consequences (see Table 3-1). Two of these consequences reflect correct decisions, two of them reflect wrong decisions. These two errors in judgment (i.e. wrong decisions) are important.

> Type I Error -This type of an error occurs whenever a hypothesis is rejected when it should have been accepted. It is also known as an alpha (α) error.

Type II Error – This type of error occurs whenever a hypothesis is accepted when it should have been rejected This is also know as beta (β) error

Table 3 1 Statistical decision errors

	Reality	
DECISION	H_0 is True	H_0 is False
Reject Hypothesis (H_0)	Wrong Decision Type I (α) Error	Correct Decision
Accept Hypothesis (H_0)	Correct Decision	Wrong Decision Type II (β) Error

From the preceding discussion it can be concluded that whenever a decision between two available alternatives is made, there is a possibility that the decision which is made will be the incorrect. This is always true unless one is completely certain about all of the factors that are involved in the decision

There is another important point to be made here. The null hypothesis in acceptance is expressed in the form 'there is no difference' between the claim, in the form of the sample average, and the design target value There is a reason for this approach The practice in construction highway acceptance is to begin with the interpretation of the situation by accepting the statement and then collecting data in order to determine whether or not the statement can be rejected. If the null hypothesis (H_0) can be rejected then the alpha (α) error (and therefore the true measure of the risk of accepting the alternative hypothesis) can be stated [Willenbrock, 1976] It should be noted that often the alternative hypothesis is the one that transportation agencies are actually interested in accepting or proving.

It is also important to realize that when a transportation agency states that they accept the null hypothesis (H_0) based on the sample data results they are really saying that they do not have the statistical evidence to reject it. However, this does not prove statistically or otherwise, that the hypothesis is correct.

Willenbrock [1976] noted that on a day to day acceptance situation, the agency representative on a project must decide whether a particular material characteristic complies with the requirements of the specifications of the project. Under a quality assurance specification, he or she will have to base the decision about a lot of material on the results of a relatively small number of tests made on the sublots of the lot. Such decisions are called statistical decisions. In attempting to reach such decisions, it is necessary to make some assumptions about the population which is involved. These assumptions are usually based upon experiments which the highway agency has conducted to determine the population mean μ and the overall population standard deviation σ of material that is currently considered to be acceptable. The assumptions that are applied to a particular lot are called statistical hypotheses. It should be understood from the beginning that, for this type of hypothesis, the only way to be certain about the truth of these assumptions is to examine the entire lot. This is often impractical or costly, and it therefore becomes necessary to take a sample from the lot and use the sample to make a decision as to whether the hypothesis is true or false.

It was stated previously in this section that if a decision is made to reject a lot of material that a contractor has produced when the material is actually satisfactory, a

Type I error is said to have been made. The risk or the probability of making such an error is generally symbolized by alpha (α) and, in the highway industry, is called the seller's risk. Seller's risk is the risk a contractor, producer, or manufacturer takes of having the highway agency incorrectly rejects material.

The other side to this coin is if a decision is made to accept a material when in actuality the material which is supplied to the highway agency is unsatisfactory; in this case, a Type II error is said to have occurred. The risk of making such an error is symbolized by beta (β) and is called the buyer's risk. Buyer's risk is the risk that the highway agency takes of accepting material that does not fully comply with specification requirements. See Table 3.2.

It is important that the ground rules (i.e. the acceptance plans) that govern the method of making such a statistical decision minimize, within practical limits, the probability of making either a type I or type II error. This aspect of acceptance plans must be fully understood since economical implications are involved. Consequently, the theory of risks must be integrated into the development of statistical acceptance plans and their related specification limits.

Usually the levels of the alpha (α) and beta (β) risks which are assigned to an acceptance plan for a particular characteristic are dependent on the importance of that characteristic to the final performance of the material. This point will be revisited later in this research.

Table 3 2 Decision risks

	Engineer's Decision (based on sample data)	
ACTUAL CONDITIONS	Reject Material	Accept Material
Material is Acceptable (H_0 is true)	Type I (α) Error Engineer Is Wrong Seller's Risk'	Engineer is Correct
Material is Unacceptable (H_0 is false)	Engineer is Correct	Wrong Decision Type II (β) Error Buyer's Risk'

Industrial quality control procedures in Juran's Handbook for Quality Control [1988] have established four classes of defects, a defect being defined as a deviation from stated tolerances. The four classes are listed below

1 *Critical* A critical defect will make the material or structure dangerous to store (or process) or to use.

2 *Major* A major defect will make the material or structure unusable, or severely impair its performance.

3 *Minor* A minor defect may impair performance but not severely

4 *Contractual* A contractual defect may have no effect on performance

Contractual characteristics are put into the specifications chiefly to permit bids to be made on a uniform basis Some modifications to the defect classifications are applied for highway applications since the above definitions were developed for material and processes not related to highways. However, from a practical and economical point of view, different levels of compliance for each class of defects are reasonable. AASHTO R-90 has indicated some recommended risk levels for each of these categories. These are shown in Table 3.3.

Table 3.3 Types of risks and their probability values

Probability Values			
Classification	Probability of Acceptance at RQL (Buyer's Risk)	Probability of Acceptance at AQL	Seller's Risk at AQL
Critical	0.005	0.950	0.050
Major	0.050	0.990	0.010
Minor	0.100	0.995	0.005
Contractual	0.200	0.999	0.001

The researcher believes that a brief example of risk analysis would be helpful at this point. This will serve as reference in latter discussions. So, in the interest of simplicity only the seller's risk is considered in the example.

Consider the hypothesis that a particular lot of concrete pavement the mean μ compressive strength is 5000psi and that the standard deviation $\sigma = 300$ psi. This implies that

H_0: $\mu = 5000$ psi (Null Hypothesis)

H_1 $\mu < 5000$ psi (let H_1 represent the Alternative Hypothesis)

The procedure that is illustrated will assume that the population follows a normal distribution The acceptance decision will be based on the average x that is obtained from a sample of size $n = 5$ from the lot If the hypothesis is true, a sample mean far removed from 5000 psi will rarely occur If the computed $\bar{x}$ is therefore reasonably close to 5000psi the hypothesis should be accepted The basic question which must be asked is what is 'reasonably close

In order to answer this question, one must refer to the sampling distribution of $\bar{x}$. From previous discussion, it is agreed that the sampling distribution can be assumed to be normal and has a standard deviation $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 300/\sqrt{5} = 134$psi

If the mean of population μ is 5000psi then the mean for the sampling distribution is also 5000psi the total range for the sampling distributions of sample means of size $n = 5$ extends from $5000 - 3(300)/\sqrt{5} = 4598$psi to 5000psi $+ 3(300)/\sqrt{5} = 5402$psi Next, the critical value of $\bar{x}$, which will cause a rejection of the lot must be computed Suppose for example that the agency decides to reject the hypothesis if x is less than -1.96σ the sample data $\bar{x}$ could be compared in order to determined if the null hypothesis (H_0) could be accepted More than $\pm = 1.96(134) = \pm 263$psi from 5000psi The values 1.96 are 'z' values It can be concluded that 95% of the $\bar{x}$ values would lie between 5000psi ± 263psi. Similarly, it would be expected that 5% of the $\bar{x}$ values would be greater or less than these limits Since this is a 'one-tailed' test, the area beneath the curve outside of the tolerance limits is 5% in the left hand tail and 0% in the right-hand tail.

When the tolerance or acceptance equation is expanded, the limiting values are obtained.

$$5000-263 \le \bar{x}$$

$$4737 \le \bar{x}$$

With this information in hand, an acceptance plan can be established as follows

1. Select a random sample of size 5 from the lot
2. Find $\bar{x}$, the mean of this lot
3. If $\bar{x}$ is greater than 4737psi accept the hypothesis that $X' = 5000$psi
4. If $\bar{x}$ does not meet the criterion in (3) reject the hypothesis, i.e. assume that $\mu < 5000$

If the plan is followed and the true population parameters are $\mu = 5000$psi and $\sigma = 300$psi, then approximately 95% of the time the correct decision will be made ie accepting the hypothesis that the mean compressive strength $\mu = 5000$. It should be noted that even if the hypothesis is true, it is expected that the wrong conclusion will be made some of the time because an μ value will be outside the limits 5% of the time even when the population mean $\mu = 5000$psi. In summary it can be said that the possible values of $\bar{x}$ which cause rejection of the hypothesis make up the rejection region. In this example the critical region of the hypothesis $\mu = 5000$psi is $\bar{x} < 4737$psi. The chance of finding a single sample which has $\bar{x}$ in this critical region if the hypothesis $\mu = 5000$psi is true as noted above is 5%. This chance is called the level of significance of the test, which is α or the seller's risk.

3.6 Operating Characteristic Curve

Once the risks of errors in arriving at decisions based on sample statistics are recognized, it becomes apparent that the objective of the agency is to minimize them In fact this is the common practice in the highway industry The popular technique used to evaluate these errors in accepting or rejecting a lot of material is one called an operating characteristic curve

The operating characteristic curve (O C.) is a graphical presentation of a sampling technique which shows the relationship between the quality of a lot and the probably of its acceptance or rejection. In reality, the O C Curve shows the probability of a type I (α) and type II (β) errors

Recall that the primary objective of acceptance sampling is to differentiate between what was defined as acceptable and what was defined as unacceptable material The O C Curve has the ability to indicate how well a given sampling plan discriminates between acceptable and non acceptable lots

Each time an acceptance procedure is used the agency must decide what will constitute an acceptable quality level (AQL) What percent defectives or poor quality material is tolerable? Ideally the agency would prefer that all materials conform to specification However it is perceived that from a realistic and economic standpoint, some small amount of material must be permitted to be less than perfect. It is also perceived that requiring 100 percent conformance would result in material prices that would be out of proportion with the anticipated increased value of the materially

It has been found that a production process in 'control' may still produce some percent of defective materials. This variability is believed to be inherent to the production process itself. Agencies normally accept materials that are within the limit of the inherent variability and reject materials that fall outside the inherent. Figure 3.6 shows an O.C. curve that sharply distinguishes the acceptable from the non acceptable material. However, this curve is not realistic. It claims to have the ability to discriminate all good materials from bad ones. It has been noted previously that this is not achievable even with 100 percent inspection. Furthermore, it has been mentioned that there will always be error and risk in any acceptance sampling technique.

3.6.1 Features of the O.C. Curve

The O.C. curve provides useful information to the highway agency representative. The vertical axis shows the probability of acceptance of a product, the horizontal axis depicts the quality measure, here the percent defective is shown because it is the chosen quality measure of the highway agency. It is recommended to enter an O.C. curve from the horizontal axis, then proceed to read vertically upwards to the curve, and then across to the Probability of Acceptance scale. In Figure 3.7 a lot which has been compacted to 95% compaction is submitted to a plan for which this O.C. curve was developed, the probability of accepting this lot is approximately 20%. The relationship between the percent compaction and the probability of acceptance has a lot more meaning than it appears in the isolated case. Because in terms of many lots, it can be concluded that if a number of lots which are compacted to 95%

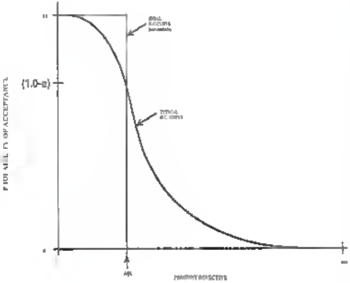


Figure 3 6 Ideal and typical operation characteristic curves

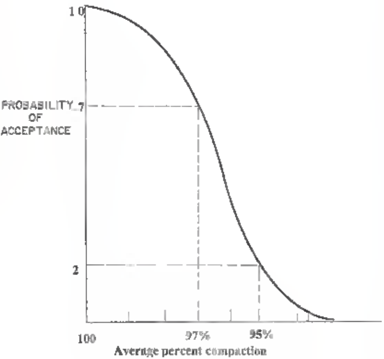


Figure 3.7 O.C. of relationship between percent compaction and probability of acceptance

compaction are submitted to this plan, approximately 20% of them will be accepted and 80% of them will be rejected. Such statements can be made about lots of any percent compaction submitted to this plan.

3.6.2 α and β Risks

Another feature of the O C curve is its ability to incorporate the buyer and seller's risks involved in evaluating the probability of acceptance of a submitted product. From Figure 3 8 the seller's alpha (α) risk is quickly identified For the percent compaction P_2 representing the acceptable quality level (AQL) the curve gives the probability of acceptance of a lot of that quality It should be noted that the probability of acceptance is equal $1 - \alpha$

Similarly for the percent compaction P_1 representing the rejectable quality level (RQL) meaning that the product is so bad that it would be detrimental to the overall performance of the pavement structure the level of the risk beta (β) given at that point is the risk of the buyer to accept a lot of that quality

The choice of alpha and beta depends upon the consequences of a product s failure to perform its intended function This is referred to as the level of criticality of the characteristic under consideration. If a product failure results in loss of life or in the complete uselessness of the unit in which the product is incorporated, it is critical failure In such cases β is normally set to approximately zero When the product s failure causes minor consequences β can be made large and α can be made smaller In the highway industry for a non-critical product, α and β are usually chosen as 0 05 and 0 10 respectively [AASHTO R 90] The true meaning of risk is how much one is willing to

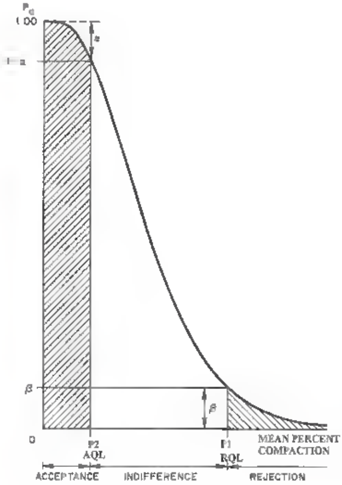


Figure 3 8 Operating characteristic curve for mean percent compaction lose in terms of dollars if an action is taken. In the section that follows, a framework that allows for cost to be explicitly considered in acceptance sampling is discussed.

3.7 Percent Defective General Theory

Among the many types of variables plan, two of them are of interest in this work. Theories underlying the known sigma and the unknown sigma plans based on percent defective are discussed here. The purpose and justification of this discussion is to establish the groundwork for the economic models that will subsequently be developed. Although the economic model which is presented in this dissertation is not based on percent defective, the theories underlying its distribution function are the same. Further the distribution function for the case of unknown standard deviation developed by Lieberman and Resnikoff [1955] can be used to develop another economic model in future research. Most general types of variables sampling plans assume that for the item qualities to be tested both the mean and the standard deviation of the measurements about their mean value are unknown. Bowker and Goode [1952] concluded that the same plans could be used for products for which the standard deviation is known, but plans, which use this known standard deviation explicitly, require smaller samples. For such products, alternative plans have been designed paralleling the unknown standard deviation plans. The sample sizes associated with the known standard deviation plans are considerably smaller than for the basic unknown standard deviation plans. The computations required to determine the disposal of a lot are simplified. Hence, these plans, although presented as alternatives, should be considered as standard for the wide variety of products. When the plans are so considered, the unknown standard deviation plans will then be reserved for products whose standard deviation can be expected to vary from lot to lot and for products whose quality history is not known In support of this approach, consider that Wallis [1947] has already concluded that if many samples of n observations are taken at random from the same population, the value of the sample statistic $(U-\bar{x})/s$ obtained from the various sample will vary So for these latter products, it is recommended that data from the initial lots to be used to determine whether or not the standard deviation is reasonably constant and if it is so an accurate estimate of its value can be determined. If the initial data shows that the standard deviation is constant, then a change can be made to a parallel known standard deviation plan. Variables sampling plans pertain to a single quality characteristic and are usually assumed that measurements of this quality characteristic are independent, identically distributed random variables. Such an assumption will be made throughout this dissertation.

For the purpose of this dissertation, sampling inspection by variables, is divided into three categories. known standard deviation plans, unknown standard deviation plans, average range plans Known standard deviation plans are based upon the sample mean and the known standard deviation. Unknown standard deviation plans are based upon the sample mean and the standard deviation Average range plans are based upon mean and the average range of the subsamples The remainder of this section is an excerpt from the Mathematical and Statistical Principles Underlying MIL-STD 414[1958] with some literal modifications

Wallis [1947] has given the background of variables acceptance sampling plans for percent defective as well as deriving approximate and exact values of the acceptability constant k and sample size *n* for the variability unknown standard deviation method-single specification limit. Wallis also gave a procedure for the double specification limits situation.

Blackwell [1947] has shown that if x has density $p_\theta(x)$, g is any unbiased estimate of θ, and $\hat{\theta}$ is a sufficient statistic for θ, then $E(g|\hat{\theta})$ is also unbiased, and furthermore has a variance no greater than that of g. Lehmann and Scheffe [1950] have extended this result and have shown that if $\hat{\theta}$ is complete, then every estimable function $h(\theta)$ possesses an unbiased estimate with uniformly small variance and this estimate is the unique unbiased estimate of $h(\theta)$ which is a function of $\hat{\theta}$. Heuristically, a statistic $\hat{\theta}$ is complete means that if there is a function of $\hat{\theta}$ which has expected value equal to zero, then this function must be identically zero for all values of $\hat{\theta}$ for which the function has positive probability.

Lehmann and Scheffe [1950] have already shown that the sufficient statistics for the normal distribution are complete. Furthermore, it is evident that there exists an unbiased estimate of the fraction of a normal population lying outside a fixed interval, e. g., the fraction of independent observations in a sample from this normal population, which lie outside this interval. Consequently, the percentage (p') of a normal population lying outside a fixed interval (L, U) is an estimable function and has an unbiased estimate with uniformly small variance and this estimate is the unique unbiased estimate of p' which is a function of the sufficient statistics.

The concern here is with estimating the parameter p' defined above. In order to eliminate the necessity of carrying the factor 100 throughout this section, the lot fraction defective is defined as

$$\frac{p'}{100} = 1 - \int_{L}^{U} \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} dz \qquad (3.7)$$

Only one of the two cases will be considered (1) μ unknown, σ known. Let $x_1, x_2, \ldots, x_n$ be independent random variables from the above normal distribution. Define p as the usual attribute estimate of the fraction defective, i.e., the ratio of the number of defective items to the sample size. It is evident that p is unbiased. Hence it follows from Blackwell [1947] and Lehmann and Scheffe [1950] results that $\hat{p} = E(\bar{p}|\hat{\theta})$ is the unique uniformly minimum variance (UMV) unbiased estimate of p/100, where $\hat{\theta}$ are the sufficient statistics for the normal distribution. Since p is the sum of the independent identically distributed random valuables taking on the values 0 and 1, it is evident that $\hat{p} = E(\bar{p}|\theta)$ is equivalent to $E(\bar{p}|\theta)$ where p is defined as follows: let y be any one of the observations ($x_1, x_2, x_3, \ldots, x_n$), say x_1, then,

$$\bar{p}(y, x_2, x_3, \ldots, x_n) = 0 \qquad (3.8)$$

if L ≤ y ≤ U and

$$\bar{p}(y, x_2, x_3, \ldots, x_n) = 1 \qquad (3.9)$$

otherwise

3.7.1 The Estimate of p' When the Population Variance σ^2 Is Known

Lieberman and Resnikoff [1950] concluded that if the observations are drawn from a normal population with an unknown mean, μ, and known variance σ^2, then

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x \tag{3.10}$$

and

$$p = E(\bar{p}|\bar{x}) \tag{3.11}$$

is a sufficient statistic. In this case equation (3.11) is the uniform minimum variance unbiased estimate of p'/100. If $\bar{p}$ is the estimate (3.8), then

$$\hat{p}(\bar{x}) = E(\bar{p}|\bar{x}) = \Pr\{\bar{p} = 1|\bar{x}\} = 1 - \Pr\{L \le y \le U\ |\bar{x}\} = 1 - \int_{L}^{U} \frac{g(y,\bar{x})}{h(\bar{x})}dy \tag{3.12}$$

where $g(y,\bar{x})$ is the joint probability density of y and x, and h($\bar{x}$) is the probability density of $\bar{x}$. Consider the joint probability density of y

and

$$\bar{x}' = \frac{1}{n-1}\sum_{i=2}^{n} x_i ,$$

$$\frac{\sqrt{n-1}}{2\pi\sigma^2} e^{-1/(2\sigma^2)[(y-\mu)^2+(n-1)(\bar{x}'-\mu)^2]}$$

The transformation

$$\bar{x} = \frac{(n-1)\bar{x}'}{n} + \frac{y}{n}$$

leads to

$$g(y,\bar{x}) = \frac{n}{2\pi\sigma\sqrt{n-1}} e^{-n/(2\sigma^2)[(\bar{x}-\mu)^2+(y-\bar{x})^2/(n-1)]}$$

and division by

$$h(\bar{x}) = \frac{\sqrt{n}}{\sqrt{2\pi}\sigma} e^{[illegible]}$$

results in

$$\frac{g(y,\bar{x})}{h(\bar{x})} = \sqrt{\frac{n}{n-1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-n(\ [illegible]}$$

Therefore

$$\hat{p}(\bar{x}) = 1 - \int_L^U \sqrt{\frac{n}{n-1}} \frac{1}{\sqrt{2\pi}\sigma} e^{-n(y- [illegible]}$$

$$= \int_{-\infty}^{\sqrt{n/(n-1)}\,(L-\bar{x})/\sigma} \phi(v)dv$$

$$+ \int_{\sqrt{n/(n-1)}\,(U-\bar{x})/\sigma}^{\infty} \phi(v)dv \tag{3 13}$$

Where $\phi(v)$ is the standardized normal density function. Finally the UMV unbiased estimate of p' is

$$p(\bar{x}) = 100\hat{p}(x)$$

3.7.2 The Known Sigma Acceptance Criterion

The acceptance procedure is formulated as follows Accept a lot of items if in the sample $p(x) \le M$ where M is so chosen that if the population percentage defective is p i.e., if the portion of the population lying outside (U,L) is p, then the probability of acceptance will be $L_{p/100}$. It is shown that in the one-sided case accept if $\bar{x} \le U - k\sigma$.

Let K_ε be defined by

$$\int_{K_\varepsilon}^{\infty} \phi(v)dv = \varepsilon \tag{3.14}$$

In the one side case, when L = -∞

$$\hat{p} = \int_{\sqrt{n/(n-1)}\,(U-\bar{x})/\sigma}^{\infty} \phi(v)dv \le M/100 \tag{3.15}$$

implies that

$$\sqrt{\frac{n}{n-1}}\frac{U-x}{\sigma} \ge K_{M/100}, \; or\; \bar{x} \le U - \sqrt{\frac{n-1}{n}}K_{M/100}\sigma$$

if one lets

$$k = \sqrt{\frac{n-1}{n}}K_{M/100}$$

the operating characteristics (OC) curves of the two acceptance procedures will be identical. For the case where there are double specification limits and the standard deviation is known, it is evident that if

$$\frac{U-L}{\sigma} < 2K_{AQL/(100)(2)}$$

the incoming value of p, the percent defective is greater than the AQL. Consequently, the lot should be rejected without a sample being drawn. The OC curve of this two sided test takes the form of a band, the lower bound of which is the one-sided OC curve. For incoming quality that is no worse than the AQL, the upper bound corresponds to the equal division of the percent defective. The band here is relatively wide, but this is desirable since this gives "good" quality a better chance of acceptance. For quality worse than the AQL, the above restriction rule (U-L)/σ eliminates certain divisions, including equal division, so that the band quickly tends to the one-sided curve.

3.8 Expectation

In statistics, a typical quantity is often necessary to describe the complete cumulative distribution function of a random variable. This suggestive quantity is referred to as the expected value of the random variable. For the continuous case, which is the one of concern in this dissertation, the expected value of the random variable X can be represented as

$$E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

The concept of expectation is important because it follows the notion of an average, which results from averaging a sequence of random variables. This average is also known as the sample mean $\overline{x}$.

The relationship between the sample mean and the expected value of any random variable, X_i, in the sequence is that as "n" gets large the sample mean approaches the expected value of the random $E(X_i)$. For the purpose of this dissertation, it is necessary to confine the discussion about expectation simply to the expectation of the random variable X. The expected value of h(X) is defined as

$$E(h(X)) = \int_{-\infty}^{\infty} h(x) f(x)\,dx$$

Thus, the expectation of the function of X is expressed in terms of the distribution of the random variable X. It should be noted that h(X) is itself a random variable. Let $X_1, X_2, \dots, X_n$ be independent variables having means, $\mu_1, \mu_2, \dots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \dots, \sigma_n^2$ respectively. Let $a_1, a_2, \dots, a_n$ be constants. Denote Y the random variable which is the linear combination of the X's i.e.

$$Y = (a_1 X_1 + a_2 X_2 + \dots + a_n X_n)$$

The random variable Y possesses three special properties. Two of them are of interest in this dissertation

1 $$E(Y) = \mu_Y = E(a_1 X_1 + a_2 X_2 + \dots + a_n X_n)$$

$$= a_1 \mu_1 + a_2 \mu_2 + \dots + a_n \mu_n$$

2 $$E(Y - \mu_Y)^2 = \sigma_Y^2 - variance(a_1 X_1 + a_2 X_2 + \dots + a_n X_n)$$

$$= a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + \dots + a_n^2 \sigma_n^2$$

This statement says that the variance of a linear combination of independent random variables is equal to the sum of the product of variances and squared constants. It can be extended to the sample mean in the following manner

Let

$$Y = x = \frac{X_1 + X_2 + \dots + X_n}{n} = \frac{1}{n} X_1 + \frac{1}{n} X_2 + \dots + \frac{1}{n} X$$

The a's are replaced with new constants. After making use of the properties defined earlier, it can be concluded that

$$E(Y) = \left(\frac{1}{n}\right)\mu_1 + \left(\frac{1}{n}\right)\mu_2 + \dots + \left(\frac{1}{n}\right)\mu_n,$$

$$\sigma_Y{}^2 = \left(\frac{1}{n}\right)^2\sigma_1{}^2 + \left(\frac{1}{n}\right)^2\sigma_2{}^2 + \dots + \left(\frac{1}{n}\right)^2\sigma_n{}^2 = \frac{\sigma^2}{n}$$

$$\sigma_Y{}^2 = \frac{\sigma^2}{n}$$

$$D = \frac{\sigma^2}{\sigma^2{}_Y}$$

Figure 3.9 shows distribution of the variance of an individual measurement and the variance of the sample mean. D is a parameter that indicates the variability of a material or product. It ranges from 2(very variable) to 50 (very stable). This distinguished it from n the sample size. However, the theory which validates D is the same as described above.

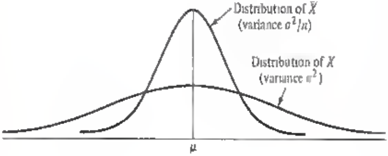


Figure 3 9 Comparison of two normal distributions with different variances

3.9 A Bayesian Framework

Representatives of the highway industry are often called upon to make decisions in the face of uncertainty; acceptance sampling is a good example. As mentioned before, the consequences of wrong decisions can be very costly. The "uncertainty" is reflected in the fact that the quantity or parameter of interest is generally unknown. Up to now, only general statistical theories underlying conventional acceptance sampling have been presented. In the first section of this chapter, the theories and principles underlying a different approach to acceptance sampling will be presented. It should be mentioned that all of the standard statistical concepts, such as mean, variance, etc., still apply in this case.

Bowker and Lieberman [1972] defined the action space as the set of all possible actions available to the decision-maker and denoted this set by the symbol "A". They also denoted by "a" the elements of the action space, that is, the individual actions. For example, if a highway engineer, as a decision maker, is looking for an aggregate supplier and only five suppliers are registered with the state, then the representative has five possible actions available: a_1, a_2, a_3, a_4, a_5, representing each supplier. In the selection process the engineer must decide how much to pay for the materials. This is referred to as a price action.

3.9.1 The State of Nature

The consequences of an action taken sometimes depend on the "state of nature" or on the particular action taken. A description of these states is given by an exhaustive (mutually exclusive) enumeration of the indices. This enumeration of the indices forms a classification of the abstracted mathematical model of the physical problem under consideration. The "states of nature" are characterized by specifying a family of probability distributions, each is represented by a possible value of the parameter of this family. Therefore, in pavement acceptance sampling, the states of nature are characterized by the class of normal distributions with unknown mean and known standard deviation. This will be discussed later.

For the purpose of this dissertation, let the set of all possible states of nature, that is, the state space, be denoted by Θ and the individual states will be denoted by θ.

3.9.2 Loss Function

It is important to note that, for each combination of action and state of nature the consequences of the action that the Highway engineer or decision-maker has taken must be measured. Therefore, the existence of a loss function $l(a, \theta)$ which measures the "loss" incurred if action "a" is taken when the state of nature is θ. This loss is usually expressed in monetary units or other reasonable utility measures. The term loss function is used to represent the consequences even though these consequences may be measured in terms of cost, gain, or profit., Measures such as gain or profit may be viewed as negative losses.

Thus, for every combination of a and θ there exists a number $l(a, \theta)$. Sometimes it is reasonable to assume that these numbers are available directly, but more often the loss incurred depends on the outcome of a random variable which depends on the true state of nature. In this case $l(a,\theta)$ is interpreted as the expected value of the loss incurred if action "a" is taken when nature is in state θ.

It should be emphasized that the losses incurred are to be considered as given numbers associated with an action and a state of nature and are not part of the decision making mechanism.

3.9.3 Criteria for Choosing among Actions

If the best action is defined to be that action which minimizes the loss, it is possible to select the best action provided that the state of nature is known. However, if the true state of nature is not known choosing the best action is not obvious. Consequently, there are some rational principles that should be followed. For example, in the case of the five aggregate suppliers referenced previously, a rational decision maker would not purchase from supplier a_4 or a_5 if he or she knows that the loss would be less if aggregates are purchased from supplier a_1, a_2, and a_3. Bowker an Lieberman stated that, in general, *an action a^* is said to be dominated by an action a if*

$$l(a, \theta) \leq l(a^*, \theta)$$

for all possible state of nature. A decision-maker should not use a dominated action. After eliminating the dominated actions, there still exists a need for a criterion to choose among the remaining actions. Alternative approaches that seem to have practical importance are the minimax principle and criteria that utilize prior knowledge about the possible states of nature.

3.9.4 Bayes Principle

If the decision maker is willing to postulate a probability distribution over the possible states of nature, *the state of nature can be treated as a random variable. the (subjective) probability distribution which describes his degree of belief about the possible states of nature is called a prior probability distribution.* By utilizing the information given by the prior distribution and the loss function, the decision-maker is able to establish a framework for choosing between alternative actions.

In general, if the state of nature, θ, is a discrete random variable, its prior probability distribution is denoted by $P\{\theta = k\} = \pi_\theta(k)$. If the state of nature, θ, is a continuous random variable. Its prior density function will be denoted by $\pi_\theta(z)$[2]

Since the state of nature, θ, in this project is considered to be a random variable, $l(a, \theta)$ is also a random variable. Therefore, a reasonable criterion for measuring the consequences of an action is to compute the expected loss $E[l(a, \theta)]$ where the expectation is taken with respect to the prior probability distribution defined over the possible states of nature, i.e.,

$$l(a) = E[l(a,\theta)] = \begin{cases} \sum_{\text{all } k} l(a,k)\pi_\theta(k) & \text{if } \theta \text{ is discrete} \\ \int_{-\infty}^{\infty} l(a,z)\pi_\theta(z)dz & \text{if } \theta \text{ is Continuous} \end{cases} \tag{3.22}$$

It should be said that, the Bayes principle calls for the decision maker to choose the action that minimizes the expected loss. this action is known as the Bayes procedure or the Bayes rule.

3.9.5 Conditional, Prior, and Posterior Probabilities

Related to the discussion on distributions, this section brings forward some important statistical notions, which play a significant part in this dissertation. In statistics, it is normally assumed that information derived from an experiment appears all at once instead of sequentially in time. In fact, partial information about the outcome of the experiment is often known or became known, and the Agency Representative could utilize this information immediately. Also, if several related random variables are of interest, information about the outcomes of some may help in predicting the outcomes of the remaining ones. This application of this concept is important in the development of the optimum sample size model because the condition placed on the random variable reduces the sample space.

Two important concepts closely related to the concept of conditional probability need to be discussed. As was discussed in the previous section, If Agency Representative is willing to postulate a (subjective) probability distribution over the unknown states of nature which expresses his or her psychological attitude, then, this information can be incorporated into the decision making mechanism for choosing among alternatives. This concept of prior distribution stemmed from the belief that the Agency Representative has accumulated experience about the possible states of nature, which is expressible in terms of probability distribution. Even though this experience is subjective in that it represent the personal beliefs of the Agency Representative, and different Agency Representatives may make different assessments, Bayes advocates conclude that it should still be used.

An updated prior distribution is called a posterior distribution. The posterior distribution of θ is defined to be the conditional distribution of θ, given that X equals x, the usual techniques for obtaining a conditional probability are used to get the posterior distribution. This posterior distribution can now be used to compute the expected loss for each action and apply the bayes principle, which calls for the Agency Representative to choose the action that minimizes the expected loss

3.9.6 Decision Procedures Theory

When acceptance sampling data is made available in the form of a random sample X1, X2, …, Xn, the highway representative or decision-maker is interested in choosing a plan or decision procedure which is dependent on the outcome of the sampling process Lieberman stated that, a *decision procedure or decision rule is a rule that specifies (1) the amount (sample size) and form of experimentation, and (2) the action that is to be taken for each possible outcome of the random sample obtained from experimentation* This decision procedure is to be chosen before the experiment is performed In order to quantify this definition, denote by X some chosen random variable which describes the "information" contained in the random sample of size *n*. The random variable X can represent such random variables as the sample mean, the sample variance, the sample median, the largest observation in the sample the random variable X_1 the vector random (X1, X2, …, Xn), and, in general, any function of the random sample. In order to relate the action a that is to be taken with each possible outcome of the random variable X define a function d such that

$$a = d[x]$$

for all possible values x that the random variable X can take on, i.e., if the random variable X takes on the value x, the decision maker takes action $a = d[x]$. Therefore, it is evident that the function d is a decision procedure.

3.9.7 The Decision-Function in Variables Acceptance Sampling

It has been assumed that the highway engineer can specify the precision that is required for a statistical procedure and that relatively low probability levels will be assigned to reduce the risks of making erroneous inferences. Often, both of these specifications will depend on the cost of sampling. What the highway engineer really wants to do is to find an economic sample size, that is, one that will maximize the net gain to be derived from an experiment. The highway engineer seeks to find that sample size such that the cost of adding an additional unit to the sample is just equal to the value of the additional information obtained. Construction of a sample-size precision schedule can assist in enabling an economic sample size to be chosen. However, a judgment decision is required, and it would be desirable to have available formulas for sample-size determination. These formulas should incorporate in them the bases for making such judgment decisions.

In this section a different approach to the problem of sample size determination is presented Mace refers to as the decision function approach. The starting point of this approach is the idea that a highway engineer performs an experiment or takes observations only to give him a basis for further action. It is assumed that the highway engineer can provide a function which relates the value obtainable from a given experiment to all possible outcomes of a particular statistical procedure, and that he can provide a function which relates the cost of the experiment to the sample size. The economic sample size is then found by maximizing in some way the expected net gain to be derived from performing the experiment and taking the appropriate action indicated.

If the probability distribution of future outcomes resulting from applying a statistical procedure is known or can be assumed at the time the sample size is determined, then a procedure for sample size determination can be designed to maximize the expected net gain (or alternatively to minimize the expected net loss) to be derived from performing the experiment and taking the action indicated. Such a procedure for sample-size determination would be called a Bayes procedure.

If information about the future outcomes resulting from applying a statistical procedure is incomplete at the time the sample size is determined and, at best, can only be expected to limit the range of possible outcomes, then a procedure for sample-size determination can be designed to minimize the maximum loss to be derived from performing an experiment and taking the action indicated. Mace [1964] referred to such a procedure for sample-size determination as a minimax procedure.

In order to accommodate both of these principles of choice, it is convenient to formulate the objective of a procedure for sample size determination in terms of minimizing losses rather than maximizing gains. Negative losses are positive gains, and positive losses are negative gains, so for Bayes procedures either formulation can be used. On the one hand, for minimax procedures it is more convenient to speak of minimizing the maximum net loss. The so called decision function approach to sample-size determination then presumes that an economic sample size is one that minimizes either the expected or the maximum net loss to be derived from performing the experiment and taking the appropriate action indicated.

The basic rationale of the decision-function approach to sample size determination can be outlined in application to an estimation problem. The aggregate loss function is regarded as the sum of two functions: a loss function due to possible imprecision in the statistical parameter being estimated and a loss function due to the cost of sampling. It is assumed that:

(1) The highway engineer can specify in dollars the loss function that can be expected as a result of a decision based on all possible errors of amount ε in the estimate of some statistical parameter, denoted by $l(\varepsilon)$.

(2) The highway engineer can specify in dollars the cost of sampling that can be expected as a result of alternative sample sizes, denoted by C(n),

(3) Random sampling is to be used such that the consequent probability distribution of errors in the estimate of the statistical parameter is known for all possible values of the sample size, denoted by $f(\varepsilon,n)$; and

(4) The highway engineer wishes to minimize the expected loss to be derived from performing the experiment and taking the appropriate action indicated (a Bayes procedure).

Then the economic sample size will be that sample size, which minimizes the aggregate loss is the sample size that minimizes

$$\int_{-\infty}^{\infty} l(\varepsilon) f(\varepsilon, n) d\varepsilon + C(n) \tag{3.23}$$

For practical work in sample-size determination, it is desirable to select a certain general class of loss functions that appear reasonable for a highway engineer to assume in a wide variety of possible situations and to present formulas based on this class of loss functions which would assist him in determining an economic sample size to a particular application. Each of the formulas presented here assumes that the cost of sampling is a linear function of the sample size

$$C(n) = C + cn \tag{3.24}$$

where C is the overhead cost and c is the unit cost of a sampling observation. The loss function that can be expected as a result of possible decisions that might be taken based on the outcome of the statistical procedure will be adapted to the nature of the broad category of problems for which systematic procedures have been developed. Moreover, for estimation problems it will be assumed that the highway engineer can and will be able to minimize his expected loss. For tests of hypothesis and selection problems it will be assumed that the probability distribution of future outcomes resulting the statistical procedure is unknown and that the Highway Engineer wishes to minimize from maximum loss.

Procedures for devising a formula for sample-size determination in estimating the mean of a variable whose distribution form is unknown but whose variance is finite can be based on the assumptions that

1) The loss resulting from a decision based on a given obtained sample mean x will be proportional to the square of the deviation of this sample mean from the true but unknown population mean , μ, that is,

$l(\bar{x}-\mu)=\lambda(\bar{x}-\mu)^2$ where λ is a known constant expressed in dollars

(2) The cost of sampling will be a linear function of the sample size, that is, C(n) = C + cn where C and c are known constants expressed in dollars.

(3) The deviation of the sample mean from the population mean will be randomly distributed with mean zero and variance σ^2/n.

(4) A preliminary estimate of the variance of individual observations of σ^2 is available

Under these assumptions the loss function will be

$$\int_{-\infty}^{\infty}\lambda(\bar{x}-\mu)^2 f(\bar{x}-\mu)dx+C+cn \tag{3.25}$$

and we seek to find that value of the sample size n which will minimize this function.

3.10 Optimization Theory and Sensitivity Analysis

Optimization techniques are used when the objective is, as it is in this dissertation, to design or plan something so that it is best in some sense. The sense in which the something is best, or is to be best, deserves important consideration on the part of the highway engineer. Often the term *performance measure* or *objective function* is used to denote that which is to be maximized or minimized.

Donald A. Pierre [1969] concluded that if performance measure and system constraints are clearly evident from the nature of a given problem, the sense in which the corresponding system is to be optimal is objective rather than subjective. But, if the performance measure and/or system constraints are partially subjective then the attainment of an optimal design is also subjective. It should be noted that in many cases, subjective criteria are more prevalent than are the objective. He also pointed out that if an objective cannot be well defined it is probably not worth much, and one can simplify the problem by eliminate it

Commonly, the form of the optimum system depends on the particular performance measure employed A performance measure is not necessarily a single entity, such as cost, but may represent a weighted sum of various factors of interest. e.g., cost, reliability, safety economy of operation and repair, accuracy of operation, size and weight, user comfort and convenience, etc

In the optimization process there could be many possible solutions that satisfy the constraints of the problem under consideration, an optimal solution is one that yields the maximum or minimum of the given performance measure Only in the most trivial cases is it feasible to test all possible solutions that satisfy the constraints of a problem to determine by comparison a solution that yields the maximum or minimum Thus, conditions are desired that can be used both to generate candidates for the optimum and to test for optimal properties. These conditions are termed conditions for optimality'

It is also important to note that optimal solutions need not be unique more than one solution may result in the same maximum or minimum. It is generally possible to find conditions that an optimal solution must satisfy, and that other, non-optimal solutions may also satisfy Such Conditions are called *necessary conditions* for optimality

A *sufficient condition* for optimality is one which, if satisfied by a given solution, guarantees that the given solution is optimum, but if a given solution does not pass a sufficient condition test it does not necessarily mean that the given solution is non optimum Only if a given optimality condition is both *necessary and sufficient* that one is guaranteed that, optimum solutions and only optimum solutions satisfy it.

Sensitivity analysis is another important aspect of optimization It is known fact that physical systems cannot be constructed or operated *exactly* according to completely deterministic specifications because of inherent limitations on the measurement of physical entities Even if one could be constructed or operated as such, it should be expected that its characteristics would change to some extent over a period of time because of environmental factors for example, temperature pressure, aging and so forth Therefore, systems can operate satisfactorily provided that certain tolerances are held by elements that influence system operation. The specification of tolerances is an essential part of any complete solution to a practical problem To specify tolerances realistically the manner in which characteristics of the system under consideration are influenced by changes in system elements should be known by the Engineer In fact, the tolerances of system elements may be selected so as to minimize the total cost of the system subject to the constraint that assured degrees of tolerances are held by essential system characteristics.

A characteristic of a system is considered to be very *sensitive* with respect to an element of the system if the characteristic is greatly influenced by relatively small changes in the element. When a part or subsystem of a system is predetermined, that is, not at the discretion of the designer, the tolerances on that part may be quite loose. With deference to this, the *sensitivity* of essential system characteristics depends on the way in which the remainder of the system is designed about the predetermined part(s). It is quite appropriate, therefore, that sensitivity considerations be included in design.

For certain types of physical systems, it is feasible to continuously or periodically monitor some of the factors upon which system performance is based. The monitored data constitute useful information if it can be processed and used to improve overall system performance. The problem of monitoring measurable entities, and of estimating from them the factors of interest, is known as the *identification problem*. A major use of on-line identification is that of keeping system performance as good as possible under changing conditions. These concepts will be revisited in chapter five. There is often a trade-off between identification and sensitivity; the more insensitive that system performance is made with respect to changes in a given parameter, the less need there is to accurately identify the parameter.

Indicators of sensitivity, *sensitivity measures*, can be based on either macroscopic or microscopic (incremental) concepts. The macroscopic sensitivity of a system characteristic is sensitivity in the large. For example, if a range of values of a particular system parameter is known to give rise to a range of values for a certain characteristic of a system, a design-center value of this system parameter may be selected at a point in the range where the system characteristic is relatively insensitive to changes in the parameter. On the other hand, incremental sensitivity of a system characteristic is applicable to small regions about some design center incremental sensitivity measures are usually based on truncated Taylor's series.

Given any system characteristic that can be represented as a function of system parameters, the fractional change in this characteristic can be estimated on the basis of fractional changes in the parameters. If the fractional changes in the parameters are known to be bounded by plus or minus certain values, worst-case sensitivities of system characteristics can be estimated.

3.11 Economic Analyses

When highway engineers evaluate pavement design alternatives, they have to consider the economic impact of each competing alternative. This is a difficult task since there are intangible values that must be considered as well. But generally, the final decision is usually based on economics. Economic analysis is the tool that makes it possible for the engineer to compare these alternatives on a level playing field. Each design is translated into its dollar value. There are several economic analysis methods used to compare alternative. For the purpose of this dissertation three of these methods are judged relevant and will be discussed.

First consider the present worth method, it is easily applicable in determining the life cycle cost of pavement or any other entity as long as the competing alternatives have equal life span, which rarely occur in practical design situation. The unequal life spans force the highway engineer to compute for a common denominator and repeat the alternatives as many time necessary to establish a common stage for comparison. The other draw back in utilizing the present worth method in life cycle analysis is that it is difficult to conceptualize that a sum that is placed ten years from now has today's dollar value. ERES [1993a] has used such an approach in their development performance related specifications. It should be noted that there are situations where the present worth is the preferred method, pavement life cycle cost is not among them. The researcher strongly recommends that they reconsider their approach.

The second method of interest here is the annual worth method. This method has the attractive features that should be considered in evaluating pavement life cycle costs. In this method each alternative is compared in terms of their annual worth. Since agencies are usually concern about annual cost of rehabilitation, repairs and maintenance for budgetary reason, this method is most suitable and practical. The values are easily understood, and there is no need to adjust for unequal life spans of alternatives.

The third technique of interest here is the benefit/ cost Analysis. Sometimes situations occur where the highway engineer has to make a decision about the cut-off point in the value of a design parameter where the optimum value cannot be explicitly determined. A good example of this is the determination of the cut–off point of the design value for the compressive strength of pavement. The nature of this parameter is such that the benefit continues infinitely, but as it the benefit increases at some point the law of diminishing return gets in the picture. Assuming all other considerations constant, the best way to determine such optimum point is through a benefit /cost analysis based on the incremental change [Collier and Ledbetter, 1988].

The equations used in calculating life cycle cost and the Benefit/Cost Analysis are provided below.

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \tag{3.26}$$

$$F = P(1+i)^n \tag{3.27}$$

$$F = A\frac{(1+i)^n - 1}{i} \tag{3.28}$$

where

P = Present worth

F = Future worth

i = Interest rate

n = Number of interest periods / or analysis period

$$\frac{B_i}{M_i} = \frac{\Delta B}{\Delta M} \tag{3.29}$$

where

B_i = Incremental benefit; obtained from subtracting the current alternative from the last one

M_i = Incremental maintenance and rehabilitation cost obtained from subtracting the current alternative from the last one

This concludes the discussion regarding fundamental principles that are essential in this research In the next chapter the modeling of the quality cost function is presented

3.12 Modeling the Cost of Quality with the Quadratic Loss Function

In the previous sections, some of the key concepts underlying the conventional acceptance sampling procedures as well as some of those underlying the alternative acceptance sampling methodology which this research is proposing were discussed. The researcher would like to point out that a number of these principles discussed are common to both methods.

This section is dedicated to the modeling of a fundamental element in this research, namely, the cost of quality with Taguchi's quadratic loss function.

The impact of costs on acceptance sampling and on quality has been previously outlined as the main thrust of this research. In addition to the many reasons already mentioned, the researcher wishes, at this point, to reemphasize the fact that conventional statistical acceptance sampling procedures are totally insensitive to cost considerations.

The quadratic loss function that will be presented in later in this section, when incorporated in the complete cost model developed in the next chapter will greatly remedy this deficiency. It should also be noted that when reference to cost of quality is made here and throughout this dissertation, life cycle cost is implied unless otherwise stated.

3.12.1 The Quality Characteristic

It is understood that, in general, every product is designed to perform some intended function. Following this maxim, at the design stage, the transportation design engineer must find some measurable characteristic or characteristics, often referred to as quality characteristics, to gauge how well a product, here a pavement, is likely to perform its intended function. For concrete pavement, four quality characteristics represented by the acronym STAR are used by the highway agency

1. Strength (Compressive)
2. Thickness
3. Air Content
4. Roughness

At this point, these quality characteristics are used without prejudice. That is, the researcher accepts them in this dissertation as if they truly represent the best or most worthy quality characteristics for concrete pavement. In Appendix B, notes on parameter and tolerance design, new criteria for selecting and evaluating quality characteristics are discussed. At that point at least the use of the compressive strength, as a quality measure will be reevaluated.

According to Taguchi [1989], the measure will possess one of the following three characteristics or types of targets, no matter how the quality of the product is measured, by a single criterion or by a combination of multiple criteria.

- the nominal the-best
- the bigger the better
- the smaller the-best

Belavendram [1995] in his book defined each of these in the following manner

1. A nominal-the-best quality characteristic is a measurable characteristic with a specific user defined target value. An example is the asphalt content in a pavement; the target might be a nominal 6%±

2. A bigger the better characteristic is a non-negative measurable characteristic that has an ideal state or target of infinity (∞). The compressive strength of concrete pavement is a good example. The higher the compressive strength the better is the quality of the pavement.

3. A smaller-the-best characteristic is a non-negative measurable characteristic that has an ideal state or target of zero. An example is the roughness or smoothness of a pavement. The smaller the roughness the better the quality of the pavement in that respect.

There is a fourth type of target that Belavendram [1995] discussed in his book on Taguchi methods. This particular type of target actually fills a gap not covered by the previous three. It is presented here because it is most appropriate for one of the concrete pavement characteristics. At first glance, one would tend to classify air content as nominal-the-best type of target, but on close view, one would quickly realize that this would not be the best classification for it. If specifications call for an air content of 6% ± say 2%, then information on strength of concrete would show different impact of air content on each side of the specifications. In other words the impact of air content is not symmetrical. The same can be said for asphalt content. In cases such as these, an asymmetrical-nominal the-best type of target is most appropriate.

3.12.2 The Quadratic loss Function

In addition to his definition of the types of targets used in Taguchi's methods, Belavendram [1995] continued to explain that for each quality characteristic there exists some function which uniquely defines the relationship between economic loss and the deviation of the quality characteristic from its target value. Belavendram also cautioned that the time and resources that are required to establish such a relationship for each quality characteristic could represent a significant investment. Taguchi [1986] found the quadratic representation of the quality loss function to be an efficient and effective way to assess the loss due to deviation of a quality characteristic from its target value, i.e., due to poor quality. A premise of Taguchi methodology is that useful results must be obtained quickly and at low cost. The use of the quadratic loss function is consistent with this philosophy.

The study of perceived quality based on the Sony America and Japan color television sets, referenced earlier in chapter 2, resulted in these findings:

1. Conformance to specification limits is an inadequate measure of quality loss.
2. Quality loss is caused by customer dissatisfaction,
3. Quality loss can be related to product characteristics,
4. Quality loss is a financial loss,
5. Quality loss function is an efficient tool for evaluating quality loss at the earliest stage of product development.

Taguchi concluded that quality is related to the loss to society caused by a product during its life cycle. A truly high quality product will have a minimal loss to society as it goes through this life cycle. In this country, it is quite popular to take a strict view of what constitutes quality. P.B. Crosby [1979] in his book titled Quality is Free supports the position that a product made according to the print, within permitted tolerance, is of high quality. Phillip J. Ross [1996] argues that this strict viewpoint embraces only the designer and the fabricator. This is referred to as the goalpost syndrome. Figure 3.10 shows a diagram representing the specification interval approach used by the highway industry and other American manufactures, often called the goalpost. This diagram depicts a situation that needs to be commented on. Point A suggest that the consumer is totally dissatisfied with the product, whereas at point B, a short distance inside the goalpost and away from point A, it suggests that the customer is completely satisfied with the product. What is missing from this philosophy, Ross added, is the consumer's requirements. A product may meet print specifications, but if the print does not meet consumer requirements, then true quality cannot be present. Ross cited that customers buy TVs with the best picture, not the one that necessarily meets specifications. The researcher believes that Ross conclusions need to be examined from a different point of view. In the highway industry, the customer (i.e., the agency) usually plays a much greater role in the decision making process to determine how a product will be designed and manufactured than the average customer. In fact, highway product specifications are developed by the highway authorities. What is interesting to note here is that although the two situations are very different, their methodology to achieve quality is quite the same. They both applied the goalpost method in their design of specifications.

3.13 Variation as a Quality Yardstick

The concept of variability has been discussed in chapters one, two, and three. In chapter one, the literature search revealed that variability of products in highway

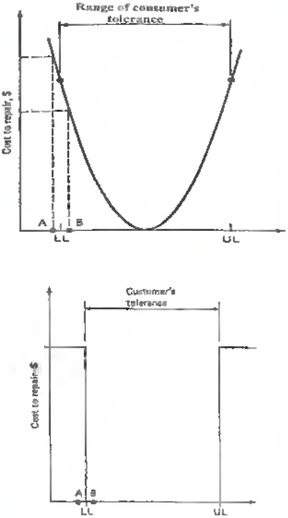


Figure 3 10 Taguchi loss function (top) and the goalpost syndrome (bottom) construction was a major contributor to early deterioration of the transportation infrastructure. In chapter two, more direct support was provided for this conclusion through the findings of the AASHTO road test. A basic statistical model was presented in chapter three and inferences drawn from this model suggest that variability was a natural phenomenon. The model further suggests that variability in measurements must be expected. The researcher strongly supports Taguchi's idea to use variability as a direct measure or yardstick to quality.

Taguchi viewed variation as a lack of consistency in the product, giving rise to poor quality. With this view, Taguchi developed methodologies aimed at reducing both elements of variation: (a) deviation from the target, and (b) variation with respect to others in the group. In Figure 3.11, a typical quality measure of a product is compared to the desired state. Note that the product mean value is off target and that the variation around the mean is large though within upper and lower acceptance limits. A much narrower distribution is desired with more frequent achievement of the target value and smaller variation around the target value.

3.14 Cost of Variation Cost of Quality

Early in his research, Taguchi [1986] concluded that variation is common to all manufacturing processes and that variation was the primary cause for rejection of parts. Parts were rejected upon inspection when they did not conform to a predefined specification. Rejection increases the cost of production. Often, as noted previously, 100 percent inspection is excessively costly or impractical; thus a defective part may reach a

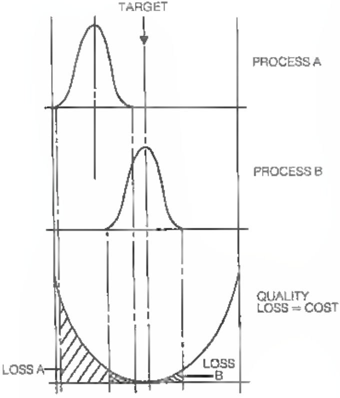

TARGET
PROCESS A
PROCESS B
QUALITY LOSS = COST
LOSS A
LOSS B

Figure 3.11 Conformance to specification and quality loss

customer and lead to warranty costs and customer dissatisfaction. Taguchi held that variation is costly even beyond the immediate factory production cost and that excessive variation causes loss of quality. He contended that the cure of quality loss is reduction of variation. Thus, he recommended that effort should be directed toward zero variation with less emphasis placed on production within fixed tolerance limits.

The quality of a product or a process may be difficult to define in quantitative terms. Quality is what the customers perceive it to be, thus quality varies from product to product and from customer to customer. The criteria customers use to judge the quality of a product are related to the satisfaction derived from the product and are numerous and often difficult to quantify. Research has shown that a lack of product consistency is a major factor in perception of poor quality. Consistency (reduced level of variation) favorably affects most common elements of quality. To customers, quality may include service after delivery, ease of assembly, and performance of a product. The Taguchi approach for reducing variation in performance is a two-step process:

1. Make the product/process perform in the best manner most of the time (less deviation from the target).
2. Make all products perform as identically as possible (less variation between the products).

Taguchi also concluded that when a product is optimal, it performs best under the available operating conditions. Depending upon the specified performance, the optimum will imply that the product has achieved the most, the least or the target value of the quality measure. Optimizing the design of a product means determining the right combination of ingredients or making the proper adjustments to the machine so that the best results are obtained.

3.15 Development of the Quality Cost Model

The quality characteristics for concrete pavement, STAR, have been classified earlier in this chapter according to their types of target as follows. The compressive strength of concrete as larger-the-better, the pavement thickness as larger-the-better, the air content as asymmetric nominal-the-best, and the roughness or smoothness as smaller-the-best.

In the interest of completeness, the mathematical formulation for each of these types of target is provided in this dissertation in terms of the average values.

Consider first the nominal-the-best type of target for a single unit of product. The researcher believes that, using the simplest and most direct type of target to lead subsequent formulations, would facilitate understanding of the methodology. If y is some variable value of a nominal-the-best characteristic, the quality loss function for y can be written as $L(y)$. The loss $L(y)$ can be expanded by a Taylor's series about the target value m:

$$L(y) = L(m+(y-m))$$

$$= L(m) + \frac{L'(m)(y-m)}{1!} + \frac{L''(m)(y-m)^2}{2!} + \cdots$$

Hence, L(m) is always constant and may be ignored since its effect is to uniformly raise or lower the value of 'y' Also since L(y) is a minimum at y = m, L'(m) = 0 The (y m)^2 is the dominant term in the above equation, larger powers being neglected

Therefore,

$$L(y) = \frac{L''(m)(y-m)^2}{2!}$$

$$L(y) \approx \lambda(y-m)^2 \qquad (3\ 30)$$

Expressed in this way L(y) is the average quality loss or loss per piece. The quadratic loss function of a nominal the best quality characteristic 'y' around the target value m is therefore given by equation 3 31 below This is the quadratic loss function for nominal the-best where

$$L(y) = \lambda(y-m)^2 \qquad (3\ 31)$$

y – the value of the quality characteristic(e.g compressive strength, smoothness, etc.),

L(y) = the loss in \$per piece of product when the quality characteristic is equal to 'y',

m = the target value of 'y'

λ – the proportionality constant or cost coefficient, which depends on the financial criticality of y

$$L(y) = \lambda(y-m)^2$$

$$\lambda = \frac{L(y)}{(y-m)^2}$$

The researcher finds it fitting, at this point, to provide a more concise explanation on the coefficient 'λ' and other parameters of the quality loss function just presented above. To accomplish this goal, an example used by Taguchi, Elsayed and Hsiang [1989] is adopted and summarized here. Shirt neck sizes are normally fabricated in Japan at 1cm intervals. So a person with a 40.5-cm neck size is forced to buy a shirt that is .5cm too small or .5cm too large. A dilemma is obvious here. The tolerance in this case is ± .5cm. If the person finds a tight shirt less bearable than a loose one, then the tolerance should be set asymmetrically on the plus and minus side in the following manner: –3cm and + 7cm, that is, if they represent the values beyond which he or she will not purchase the shirt. To grasp the functionality of the coefficient 'λ' and to calculate it, one needs to know the cost associated with exceeding these tolerance limits, namely –3cm and + 7cm. Let them be represented from now on by Δ_1 and Δ_2, respectively, with the understanding that Δ is the general symbol for tolerance. The coefficient 'λ' is the ratio of the cost of exceeding the tolerance to the square of the tolerance. Let A_0 represent the average cost of exceeding the tolerance, when the tolerance is Δ.

Hence,

$$\lambda = \frac{A_0}{\Delta^2} \tag{3.32}$$

$$L(y) = \frac{A_0}{\Delta^2}(y - m)^2$$

The quality loss function described above is used for assessing the quality loss for one piece of product. More often, it is needed to evaluate the average quality loss of a product based on a sample of a product. Hence, if a sample of product is taken, there will be many values of y, represented as y_i, and the average of such a sample may be denoted y. Of course y may or may not be equal to the target value m. The average quality loss can be formulated from equation (3.31) as

$$
\begin{aligned}
MSD &= \frac{(y_1 - m)^2 + (y_2 - m)^2 + \quad + (y_n - m)^2}{n} \\
&= \frac{1}{n}\sum_{i=1}^{n}(y_i - m)^2 \\
&= \frac{1}{n}\sum^{n}(y^2 - 2y\, m + m^2) \\
&= \frac{\sum y^2 - 2m\sum y_i + \sum m^2}{n} \qquad (3.33)
\end{aligned}
$$

The term in brackets (above) is the average of all the values of $(y - m)^2$ and is called the mean squared deviation (MSD). More specifically it may be referred to as the mean squared deviation from the target and is mathematically equivalent to substituting

$$
\begin{aligned}
\sum y_i &= n\bar{y} \\
\sum m^2 &= nm^2
\end{aligned}
$$

$$MSD = \frac{\sum y_i^2}{n} - \frac{2mn\bar{y}}{n} + \frac{nm^2}{n}$$

$$= \frac{\sum y^2}{n} - 2my + m^2$$

To simplify this equation further, we add $(-y^2 + y^2)$ into the last equation and regroup the terms as follows

$$MSD = \frac{\sum y_i^2}{n} - 2my + m^2$$

$$= \frac{\sum y^2}{n} - \bar{y}^2 + \bar{y}^2 - 2m\bar{y} + m^2$$

$$= \left[\frac{\sum y^2}{n} - \bar{y}^2 \right] + \left[\bar{y}^2 - 2m\bar{y} + m^2 \right]$$

$$= \left[\frac{\sum y_i^2}{n} - \bar{y}^2 \right] + \left[\bar{y} - m \right]^2$$

$$= \sigma_n^2 + \left[\bar{y} - m \right]^2 \qquad (3.34)$$

Where σ_n is usually called the *population standard deviation* and $(y - m)$ is the bias of the sample from the target. For practical reasons, however, σ_{n-1} is used. It is usually called the *sample standard deviation*. Since

$$L(y) = \lambda\left[MSD \right]$$

substituting [*MSD*],

$$L(y) = \lambda\left[\sigma_n^2 + (\bar{y} - m)^2 \right] \qquad (3.35)$$

It should be noted that the quality loss function always determines the loss per piece. Therefore, the total loss can be calculated simply by multiplying by the total number of products.

Having developed the nominal the-best the smaller-the-better quality loss function can be developed rather easily In fact, the smaller-the best quality loss function can be extrapolated directly from the nominal-the-best- quality loss function. Referring to equation 3 35, the quality loss function was given as

$$L(y) = \lambda\left[\ MSD\ \right]$$

$$L(y) = \lambda\left[\ {\sigma_n}^2 + (y - m)^2\ \right]$$

A smaller the-best characteristic can never take a negative value. Thus it is a one sided quality loss function because y cannot take on a negative value. The ideal value is zero and as this value increases, the quality loss becomes larger For a smaller-the-best characteristic, the quality loss function for many pieces can be derived from the above equation Since the target for the smaller-the-better characteristic $m = 0$ the quality loss function becomes

$$L(y) = \lambda\left[\ {\sigma_n}^2 + (\bar{y} - m)^2\ \right]$$

$$= \lambda\left[\ {\sigma_n}^2 + (\bar{y} - 0)^2\ \right]$$

$$= \lambda\left(\sigma^2 + \bar{y}^2\right) \qquad (3\ 36)$$

Again it must be remembered that $L(y)$ represents the average loss per piece of product, estimated from a sample of products with a sample standard deviation σ and a mean y

The larger the-best quality loss function can mathematically be regarded as the inverse of a smaller the-best characteristic. Just as for the smaller the-best quality loss function, the larger the-best quality loss function cannot take a negative value. Zero is the worst value and as the quality characteristic value becomes larger, the performance becomes better that is, the quality loss becomes smaller. The ideal value is infinity at which the loss is zero. A special case of the larger-the-best characteristics exists where a characteristic can take a range of values such as 0% to 100% such characteristic is a pseudo larger the-better characteristic and may be regarded as a larger-the-better characteristic for most practical purposes. The larger the-best quality loss function can be found by averaging the quality loss for one piece ($L(y)=\lambda(1/y)^2$)

$$L(y)=\lambda\frac{1}{n}\left[\frac{1}{y_1^{\;2}}+\frac{1}{y_2^{\;2}}+\dots+\frac{1}{y_n^{\;2}}\right]$$

$$=\lambda\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{y_i^{\;2}}\right]$$

$$=\lambda\left[MSD\right] \tag{3.37}$$

Notice that the mean squared deviation [MSD] is dependent on the target value of the quality characteristic. While [MSD] for nominal the best and smaller the best characteristics can be calculated relatively easily, calculation of [MSD] for the larger the-best characteristic requires the use of Taylor s theorem. In general,

$$f(y)=f(\mu+\ \ (y-\mu))$$

where μ is the population mean and can be expanded by Taylor's expansion

$$f(y)=f\frac{(\mu)}{0!}+f'\frac{(\mu)(y-\mu)}{1!}+f''\frac{(\mu)(y-\mu)^2}{2!}+f'''\frac{(\mu)(y-\mu)^3}{3!}+\dots \qquad (3.38)$$

where f', f'', and f''' are the first second and third derivatives of f(y), and higher derivatives have very little significance. The equation can be rewritten as

$$f(\mu)\approx f(\mu)+f'(\mu)(y-\mu)+f''\frac{(\mu)(y-\mu)^2}{2} \qquad (3.39)$$

let

$$f(y)=\frac{1}{y^2} \qquad (3.40)$$

differentiating with respect to *y*

$$f'(y)=\frac{-2}{y^3}$$

$$f''(y)=\frac{6}{y^4}$$

so that

$$f(\mu)\approx f(\mu)+f'(\mu)(y-\mu)+f''\frac{(\mu)(y-\mu)^2}{2}$$

$$=\frac{1}{\mu^2}+\frac{-2}{\mu^3}(y-\mu)+\frac{6}{\mu^4}\frac{(y-\mu)^2}{2}$$

$$=\frac{1}{\mu^2}+\frac{-2}{\mu^3}(y-\mu)+\frac{3}{\mu^4}(y-\mu)^2$$

The expected value of this equation is

$$E(f(y)) \approx E\left(\frac{1}{\mu^2}\right) + \frac{2}{\mu^3}E(y-\mu) + \frac{3}{\mu^4}E(y-\mu)^2$$

$$= \frac{1}{\mu^2} + \frac{-2}{\mu^3}(0) + \frac{3}{\mu^4}\sigma^2$$

$$= \frac{1}{\mu^2} + \frac{3\sigma^2}{\mu^4}$$

$$= \frac{1}{\mu^2}\left[1 + \frac{3\sigma^2}{\mu^2}\right] \tag{3.41}$$

Since the expected value of (y - μ) is zero and the expected value of (y - μ)^2 is the variance. It follows that the mean squared deviation can be written as

$$MSD = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{y_i}\right)^2$$

$$\approx \frac{1}{n}\frac{n}{\mu^2}\left[1 + \frac{3\sigma^2}{\mu^2}\right]$$

$$\approx \frac{1}{\mu^2}\left[1 + \frac{3\sigma^2}{\mu^2}\right]$$

Thus the quality loss function for the larger-the-best characteristic can be written as

$$L(y) = \lambda\left[MSD\right]$$

$$L(y) \approx \frac{\lambda}{\mu^2}\left[1 + \frac{3\sigma^2}{\mu^2}\right] \tag{3.42}$$

In certain situations, deviations from the quality characteristic nominal the-best may be different on either side of its tolerance. In such cases, an asymmetric nominal the-best quality loss function is used with different cost coefficients λ_1 and λ_2. So the total quality loss is approximated using the following asymmetric quality loss function.

$$L(y) = \lambda_1(y-m)^2 \quad y \leq m$$

$$L(y) = \lambda_2(y-m)^2 \quad y > m \tag{3.43}$$

See Figure 3.12 for graphs of quality characteristics. The population parameters (μ and σ) can be estimated from a sample of product with mean y and a standard deviation σ_{n-1} or S.

In the subsequent chapter, the quality loss function for the nominal type of characteristic is used in developing the economic model for the optimum sample size.

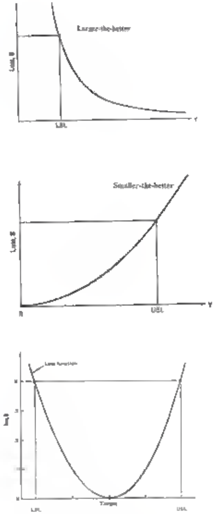


Figure 3-12 Larger the better, nominal the best, smaller the better

CHAPTER 4

MODELING THE OPTIMUM ACCEPTANCE PLAN

The work presented so far has been largely limited to the gathering of background materials, concepts and techniques that the researcher found indispensable in the process of developing the optimum acceptance sampling methodology, which is the primary concern to this dissertation. In the last chapter, it was concluded that each quality characteristic could be associated with some function, which uniquely relates its economic loss to its deviation from target value. The conclusion that this function can efficiently and effectively be replaced by a quadratic loss function constitutes the basis for this chapter. Also in the last chapter, the quality cost models for different quality characteristics, namely, nominal-the-best, the larger-the-better and smaller the better were presented. The cost model derived for the larger the better was first used to develop the optimum sampling policy because it best represents the quality characteristic of concern, this approach did not produce the intended result in spite of all efforts. Another approach had to be taken, the larger-the-better model and comments are available in appendix A. The researcher concluded that Belavendram's [1995] suggestion that the quality loss function is equal to λ(MSD) is valid only in situations where the objective is parameter or tolerance design. This research does not focus on parameter or tolerance design. This insight resulted from observing the behavior of the failed procedure described above. It is to be noted that the literature is not clear as to the range of application of the quality cost functions.

One of the problems encountered in the development of the model with the-larger the-better quality characteristic is that it does not possess the statistical properties that are inherent in the nominal the-best and smaller the-better quality characteristics cited in chapter three. Therefore the mathematical formulation became very complicated and cumbersome. The problem was not discovered until the modeling was completed. Once completed, the researcher proceeded with the optimization process of the model, and the resulting first partial derivative was greater than zero. The function was always increasing, yielding no maximum or minimum point. So the conditions for optimality could not be satisfied. In fact this should have been obvious from examining the loss function closely. It was apparent then that there was also a conceptual problem in applying this quality loss function for the purpose of acceptance sampling. In economic acceptance sampling using the Taguchi approach, the concept aims at accounting for the loss to society due to deviation from the target value. This point has been made in several occasions in this dissertation. However, what was not so clear is the fact that this loss to society concept is independent of the quality characteristic being measured. It follows that this loss to society, to be properly modeled, only requires the use of a quadratic function. It is recommended that the highway agencies in conducting parameter design use the quality cost models for the other types of quality characteristics presented in the preceding chapter.

The researcher pointed out earlier that acceptance sampling by attributes is the preferred acceptance sampling method of main stream industry, and that the highway industry has demonstrated its preference for acceptance sampling by variables through its almost exclusive use of it in their projects. However, up to now, there has not been any effort toward developing acceptance sampling with explicit economic considerations, in spite of the obvious need. Even though main stream industry has shown preference for acceptance sampling by attributes, it also has recognized that in some situations sampling by variables is most appropriate for reasons cited earlier. Because of this limited use of variables sampling methods, only few economic models dealing directly with sampling by variables have been found.

4.1 Development of the Model

It is a known fact that in an acceptance-sampling plan, the key element is the determination of the sample size. The purpose of this chapter is to present and analyze an economic sampling model with the ability of discerning the optimum sample size explicitly. Once formulated, an optimization procedure is followed to obtain the optimum desired values in terms of the upper and lower limits and the sample size. Boundary conditions are set to facilitate the task of the decision maker. The model is analyzed for sensitivity to changes in parameters. In addition, a procedure for risk assessment is presented and discussed. Also an example, using data provided by Richard Weed of the New Jersey Department of Transportation and Edward Racila of Law Engineering, is provided.

In this dissertation, two cost models are presented for acceptance sampling by variables. The first model is for the two-sided case, and the second is for the one-sided case. It is assumed that lots failing to meet the specifications are rejected. Conditions for optimal sampling plan development are presented for both models and are illustrated by example

The decision variables are the sample size, n, upper control limit, U and lower control limit, L, for the sample mean. When the sample mean, $\bar{x}$, based upon a sample size, lies, (as determined by hypothesis testing) between the lower and upper control limits the lot is accepted Otherwise, it is rejected.

4.1.1 Model Assumptions

The researcher concludes that an economic model must encompass all tangible costs that occur in the life cycle of the product, and the quality loss is a function of the variability that exists in the product with respect to the optimum target value. In light of these conclusions, the model presented here follows a cost structure that includes the life cycle cost of the product as well as the sampling costs These costs can be subdivided into three categories as discussed below The model assumes that the variance is known or can be estimated. It is recognized that, ultimately a model that assumes unknown variance has to be developed to better respond to the needs of the highway industry where, in some situations, especially in the cases of new products, the variances are not known. However this was not possible within the confines of this dissertation. It is expected that the model presented will make the future task easier to accomplish. The probability distribution function for the case of unknown variance developed by Lieberman and Resnikoff [1955] can be used to develop such a model, which in reality will be an extension of this model. It is important to note that in the construction industry the unknown variance approach is used for a reason that is not always cited or made clear. That is, since a lot quality is measured and accepted on a lot by lot basis, the percent defective procedure for known variance provided by Lieberman and Resnikoff is inadequate when between lot variation is a concern. But if the highway agencies opted to take full advantage of Bayesian modeling by defining a prior distribution, and the variances can be reasonably estimated, then the model presented here is quite sufficient.

4.1.2 Cost Components

The first cost element considered in the model is the cost of *Inspection*, which it includes two components. The first is unit cost of sampling inspection, C_i, which includes all direct costs attributed to the inspection of an individual sample in a lot. The total cost of this particular component is a linear function of the sample size. The other component is the fixed inspection cost, C_s, which includes all direct and indirect charges that result from the existence of sampling inspections that are independent of the sample size, n.

The second cost element considered is the cost of *Acceptance*. To the highway agency, this is the most important of all the cost components since this cost is borne by the highway agency since it occurs after acceptance. When a production lot is accepted, the entire lot of size N is treated as being of sufficiently high quality to be used for its intended purpose. However, lot acceptance does not necessarily imply that the lot is defect free. Hence, it is assumed that each defective unit or sublot in the accepted lot leads to a cost C_A. It includes any costs (such as increase in rehabilitation and maintenance, user cost, and ultimately early final product failure) which may arise when an attempt is made to use a defective unit for its intended purpose Such an increase is the result of a type II error

The third cost element is the cost of *Rejection.* When a production lot is rejected as a result of sampling inspection, the lot is screened (re-tested with additional samples) to confirm the lot quality See Figure 4.1 for control chart. In addition lots that are totally rejected are replaced at a cost C_R It includes all charges for equipment, labor, and materials necessary to replace the pavement. As well as all costs associated with the disposal of the defective unit, and the user cost (to be determined by the agency) if any This cost is borne by the contractor (since it occurs prior to acceptance) and it is a result of a type I error

Schmidt, Case, and Bennett [1980] noted that, as a result of sampling inspection (quality control), the production lot may be either accepted or rejected. Following Mace [1964], Moskowitz and Tang [1992] and Tagaras[1994] the expected cost of quality control given that the production lot is accepted and has deviation of the mean, μ, from the target value is given by summing the three cost components

Expected Total Costs of Inspection = Expected (Cost of Inspection + Cost of Accepting a Defective Lot + Cost of Rejecting a Good Lot)

$$ETCI = ECI + ECA + ECR \qquad (4.1)$$

where the inspection cost is equal to the sum of the fixed inspection cost due to sampling and the variable cost of testing an individual unit or sample

Cost of Inspection = $C_s + C_i n$

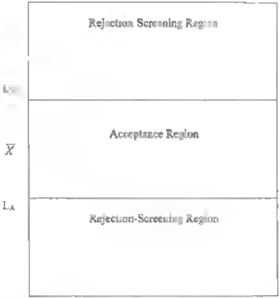

Rejection Screening Region
Acceptance Region
$\overline{X}$
L_A
Rejection-Screening Region

Figure 4.1 Control Chart for Mean where an Inspection Lot may be Accepted, Screened Accepted or Rejected

For the two-sided case the probability of finding a unit which conforms to an acceptable quality level within a lot with deviation of the mean μ is given by equation

$$P(\text{conforming unit}|\text{acceptance}, \mu) = \int_{S_L}^{S_U} f(x|\mu)dx \qquad (4.2)$$

Conversely, The probability of finding a sublot which does not conform to the acceptable quality level within a lot with deviation of the mean μ, and is however accepted is given by equation

$$P(\text{defective unit}|\text{acceptance}, \mu) = 1 - \int_{S_L}^{S_U} f(x|\mu)dx \qquad (4.3)$$

Equations 4.3 and 4.4 provide the basis for probabilistic formulation of the cost model using a single sublot. From this point on, the lot average shall be used for analysis instead of the individual sublot. This is more practical since lots are accepted based on average samples taken from each sublot. Lieberman and Resnikoff [1955] concluded that in the case of known variance, the sample mean is a sufficient statistic for the lot. Equation 4.3 and 4.4 can then be replaced with

$$P_a(\mu) = \int_{L}^{U} f(\bar{x}|\mu)d\bar{x} \qquad (4.5)$$

and

$$P(\mu) = 1 - \int_{L}^{U} f(\bar{x}|\mu)d\bar{x} \qquad (4.6)$$

For the one sided case, change the limits in equation 4.5and 4.6 to L to ∞

Equation 4.5 and 4.6 give the probability of acceptance and the probability of rejection of the lot based on the sample mean $\bar{x}$ respectively

From chapter 3 section 3.12, it was demonstrated that the quality loss function, presented by Taguchi [1984] could be used to determine the expected loss in quality of a product based on it variability from a target value. It is appropriate at this point to apply the loss function equation to the model developed in this chapter. Recall that the expected value of any function g(x) of the random variable 'x' is the weighted average of the function (weighted by the probability of its occurrence) over all possible values of the variables

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx \tag{4.7}$$

Recall that in chapter four, it was also shown that

$$L(y) = \lambda(MSD)$$

$$L(y) = \lambda(y - m)^2 \tag{4.8}$$

It follows that, with appropriate substitutions for *y* and *m* from the derivation given for more than one sample (the average of samples) in a lot is of interest.

The above equation can be rewritten as below neglecting the middle term

$$L(x) = \lambda(\mu^2 + \sigma^2) \tag{4.9}$$

Following Tang and Tagaras, the cost of acceptance of an inspection lot with deviation μ of its target value μ_0 is

$$C_A(\mu) = \left[\int_{-\infty}^{\infty} L(x) f(x|\mu) \right] P_a(\mu) \tag{4.10}$$

given that

$$g(\bar{x}|\ \mu) = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}} e^{\frac{-n(\bar{x}-\mu)^2}{2\sigma}}$$

$$P_a(\mu) = \int_L^U g(\bar{x}|\mu)d\bar{x} = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}} \int_{-W(n)}^{W(\mu)} e^{-\frac{n(\bar{x}-\mu)}{2\sigma^2}} d\bar{x}$$

hence

$$\int_{-\infty}^{\infty} L(x) f(x|\mu)dx = \lambda E(x^2|\mu) = \lambda(\mu^2+\sigma^2)$$

hence the expression above can be rewritten as

$$C_A(\mu) = \lambda(\mu^2+\sigma^2)Pa(\mu) \tag{4 11}$$

Taking the expectation of $C_A(\mu)$ with respect to μ leads to the following expression for the expected cost of acceptance per lot

$$C_A = \lambda \int_{-\infty}^{\infty} (\mu^2+\sigma^2)Pa(\mu)h(\mu)d\mu$$

$$C_A = \lambda \int_{\infty}^{\infty} \mu^2 Pa(\mu)h(\mu)d\mu + \lambda\sigma^2 \int_{\infty}^{\infty} Pa(\mu)h(\mu)d\mu \tag{4 12}$$

where

$$h(\mu) = \frac{\sqrt{D}}{\sigma\sqrt{2\pi}} e^{\frac{\mu^2}{2\sigma^2\ \ldots}}$$

The probality density function of 'x', which is the deviation of a quality characteristic of an individual unit from the target value μ_0 Since $f(x \mid \mu)$ is assumed to have a normal probability distribution with mean μ and variance σ^2, and $g(\bar{x} \mid \mu)$ which has been defined as the probability density function the sample mean, is also normal with mean μ and variance σ^2/n Equation 5 5 can be simplified to

$$P_a(\mu) = \phi([U-\mu]\sqrt{n/\sigma}) - \phi([L-\mu]\sqrt{n/\sigma}) \qquad (4\ 13)$$

$$P_a(\mu) = 1 - \phi([L-\mu]\sqrt{n/\sigma}) \qquad (4\ 13b)$$

where $\phi(\,)$ is the cumulative standard normal function.

Use equation 4 13b for the one-sided case

Taking the expectation of $C_R(\mu)$ with respect to μ leads to the following expression for the expected cost of acceptance per lot

$$C_R = c \int_{-\infty}^{\infty} [1 - Pa(\mu)]h(\mu)d\mu$$

$$= c_r - c_e \int_{-\infty}^{\infty} P_a(\mu)h(\mu)d\mu$$

$$(4\ 14)$$

$$= c \;\; - c \;\; P_a$$

Where P_a is the probability density of acceptance, calculated from

$$P_a = \int_{-\infty}^{\infty} Pa(\mu)h(\mu)d\mu \qquad (4\ 15)$$

$$P_n = \frac{\sqrt{\frac{nD}{(n+D)}}}{\sigma\sqrt{2\pi}} \int_{-U(n)}^{U(n)} e^{\frac{-nDx^2}{2(n+D)\sigma^2}} dx \qquad (4\ 15b)$$

$$P_a = \frac{\sqrt{\frac{nD}{(n+D)}}}{\sigma\sqrt{2\pi}} \int_{-U(n)}^{\infty} e^{\frac{-nDx^2}{2(n+D)\sigma^2}} dx \qquad (4\ 15c)$$

Use equation 4 15c for the one-sided case (L=-U(n)) The total cost per inspected lot for a given sample size n and upper and lower limits, U, and, L, respectively, is obtained from the equation given below

$$ETCI(n,U,L) = C_I + C_A + C_R$$

$$= c_s + c\,n + Nc_n + N\lambda \int_{-\infty}^{\infty} \mu^2 P_a(\mu) h(\mu) d\mu + N\lambda\sigma^2 P_a \quad Nc_n P_a \qquad (4\ 16)$$

$$ETCI(n,U(n)) = c_s + c\,n + c_r + \frac{N}{\sigma}\sqrt{\frac{nD}{(n+D)2\pi}}\left(\lambda\sigma^2 + \frac{\lambda\sigma^2}{D} - c_n\right)\int_{-U(n)}^{U(n)} e^{\frac{-nDx^2}{2(n+D)\sigma^2}} dx \qquad (4\ 16b)$$

$$ETCI(n,U(n)) = c_s + c\,n + c_n + N\left(\lambda\sigma^2 + \frac{\lambda\sigma^2}{D} - c_r\right)(1 - \int_{-\infty}^{\frac{U(n)}{\sqrt{\left(\frac{\sigma^2}{D} + \frac{\sigma^2}{n}\right)}}} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx \qquad (4\ 16c)$$

Use equation 4 16c for the one-sided case The expected total cost per lot in the case of acceptance without sampling, ETCA is calculated from the following equation

$$ETCA = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} L(x) f(x|\mu)dx\right]h(\mu)d\mu$$

$$= \lambda \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x^2 f(x|\mu)dx\right]h(\mu)d\mu$$

$$= \lambda \int_{-\infty}^{\infty}(\sigma^2 + \mu^2)h(\mu)d\mu$$

$$= \lambda(\sigma^2 + \mu^2) \tag{4.17}$$

Since the expected value of μ, $E(\mu)=0$ using $\sigma_\mu^1=\sigma^2/D$, then ETCA can be written as

$$ETCA = \lambda\sigma^2\left(1-\frac{1}{D}\right) \tag{4.18}$$

In the case of rejection without sampling, the expected cost per lot, ETCR, is given by

$$ETCR = C_r = c_r \tag{4.19}$$

4.2 Model Optimization

In this section, optimization techniques are applied to obtain the best solution for the given constraints. The first and second derivative tests for locating relative maxima and minima are performed on the objective function of equation 4.16. Through this step, the minimum cost of sampling, the optimum sample size, and the economic tolerance will be determined. Recall that in the model, cost is the performance measure(i.e. the sample sizes which result from the lowest ETCI)

$$\frac{\partial ETCI(n,U)}{\partial U} = 0$$

Now substituting

$$\frac{\partial}{\partial U}\left[c_s + c_r + \lambda\int_{-\infty}^{\infty}\mu^2 P_e(\mu)h(\mu)d\mu + \lambda\sigma^2 P_e - c_r F \text{[illegible]}\right]$$

since

$$\frac{\partial P_e(\mu)}{\partial U} = \frac{\partial}{\partial U}\int_{-U}^{U} g(\bar{x}|\mu)d\bar{x} = g(U|\mu) + g(-U|\mu)$$

the first order condition becomes

$$\lambda\int_{-\infty}^{\infty}\mu^2\left[g(U|\mu) + g(-U|\mu)\right]h(\mu)d\mu = (c_r - \lambda\sigma^2)\int_{-\infty}^{\infty}\left[g(U|\mu) + g(-U|\mu)\right]h(\mu)d\mu$$

$$\int_{-\infty}^{\infty}\mu^2\left[g(U|\mu) + g(-U|\mu)\right]h(\mu)d\mu = 2\psi(\mu)\left[\frac{n^2U^2}{(n+D)^2} + \frac{\sigma^2}{n+D}\right] \tag{4.20}$$

$$\int_{-\infty}^{\infty}\mu^2\left[g(U|\mu) + g(-U|\mu)\right]h(\mu)d\mu = \psi(\mu)\left[\frac{n^2U^2}{(n+D)^2} + \frac{\sigma^2}{n+D}\right] \tag{4.20b}$$

$$\int_{-\infty}^{\infty}\left[g(U|\mu) + g(-U|\mu)\right]h(\mu)d\mu = 2\psi(U) \tag{4.21}$$

$$\int_{-\infty}^{\infty}\left[g(U|\mu) + g(-U|\mu)\right]h(\mu)d\mu = \psi(U) \tag{4.21b}$$

use equation 4 20b and 4 21b for the one-sided case where the first order condition becomes

$$\lambda\left[\frac{n^2U^2}{(n+D)^2}+\frac{\sigma^2}{n+D}\right]-c_r-k\sigma^2$$

Then, from the first order condition, the optimum U for each given sample size *n* can be obtained

$$U(n)=\sqrt{\frac{\left[c_r(n+D)-(n+D+1)\lambda\sigma^2\right](n+D)}{\lambda n^2}} \tag{4 22}$$

The second derivative of ETCI (n, U) with respect to U is as follows

$$\frac{\partial ETCI(n,U)}{\partial U^2}=\frac{\partial}{\partial U}N\left[2\psi(U)(\lambda\left[\frac{n^2U^2}{(n+D)^2}+\frac{\sigma^2}{n+D}\right]-(c_r-\lambda\sigma^2))\right]$$

$$=(N)\left[2\psi(U)\frac{2\lambda n^2U}{(n+D)^2}+2\psi'(U)(\lambda\left[\frac{n^2U^2}{(N+D)^2}+\frac{\sigma^2}{n+D}\right]-(c_r-\lambda\sigma^2))\right]$$

when U(n) is given by the first order condition 4 22 the second summand above vanishes and we have

$$\frac{\partial ETCI(n,U)}{\partial U^2}=2\psi(U)\frac{2\lambda nU}{(n+D)^2}\geq 0$$

4.3 Sensitivity Analysis

From equation 4 22 an important boundary condition is derived. It implies that if

$$c_r<\frac{n+D+1}{n+D}\lambda\sigma^2 \tag{4 23}$$

for any given sample size n then the decision must be to simply reject inspection lots rather than use a sampling plan with sample size n The limiting case is when

$$c_r = \frac{n+D+1}{n+D}\lambda\sigma^2$$

where the limit U equal to 0 and the lot is accepted if and only if the sample mean is exactly equivalent the target value As stated previously, if the cost of rejection is less than the cost of variability of the lot, then the boundary condition equation given above holds for every sample size n and the optimal decision is to simply reject the inspection lots. It follows then, that the expected total cost of rejection will also be lower than expected total cost of acceptance as described in equations 4 19 and 4 18 respectively If equation 4 22 is examined further, several intuitive and interesting insights can be obtained Note that for any given sample size n U(n) is decreasing in σ and λ and increasing in c_r and D see tables 4 2-4 11

Each of the parameters in the cost model does play a role in defining the optimal acceptance plan. For example if the fixed cost (cs) and the variable cost (CI) (when included) of sampling inspection are high relative to the remaining cost components, the optimum decision might be to accept all lots without sampling inspection. However, if inspection costs are low and the cost of rectifying defects detected during inspection is also low relative to the cost of accepting a defective product, the optimum decision policy may be to screen all lots rather than subject them to sampling inspection. It should be noted that the last suggestion although not impossible, is not practical in pavement sampling inspection It is not expected that the unit costs included in the model will completely determine the optimum policy regarding the treatment of inspection lots The cost of an accepted defect can be quite high tending to indicate that the lot should be completely screened without sampling inspection. But if the variance of the lot mean and the variance of product (very stable material) dimension within a lot are quite small, the optimum decision may be to invoke a screening or scrapping acceptance sampling plan or to accept all lots without inspection. It follows that the likelihood of occurrence of a lot with anything but a very small fraction of defective unit is low. Again, this theoretical approach may not be applicable in pavement sampling. For this reason it is proposed to apply a double sampling inspection in lieu of screening the lot.

The parameter D has been defined as the ratio of the variance of x to the variance of μ. Its effect on the relative optimal sampling plan is very important and should be considered. For small values of D, the optimum plan tends to call for scrapping rejected lots, while as D increases the optimum acceptance sampling plan changes to one calling for screening or rejected lots. This result can be interpreted as follows. Very small values of D imply that the lot mean, μ, is varying significantly from lot to lot. Hence, one is likely to find lots which contain a large fraction of defective products. For large D, the lot mean, μ, is rather stable from lot to lot. In that case, even rejected lots are not likely to contain enough defective items to warrant the replacement of the entire lot.

Consider the integrals in equation 4.20 are always positive; and a solution exist if and only if

$$c_s - \lambda\sigma^2 > 0$$

although if

$$c_r \leq \lambda\sigma^2$$

then, rejection without sampling is more economical than any sampling plan. In the illustrative example it will be assumed that $c_r - \lambda\sigma^2 > 0$

Both the cost of acceptance and the cost of rejection have been introduced and defined in section 4.1.2. It is fitting at this point to revisit these definitions in order to generate reasonable values to be used in the example. c_r and λ are the cost of acceptance parameters respectively. The cost of rejection shall be considered first. It should be noted that in the development of the cost model, the lot size N was assumed to be equal to one therefore, the unit cost of rejection c_r and C_r are equivalent. For the example presented, the cost of rejecting a lot will be assumed to be the same as the bid price for the lot. This cost is not expected to result from life cycle analysis in actual situations. However, as mentioned previously, the parameter λ is a product of life cycle analysis.

4.4 Example

In this section, an example is presented in order to evaluate the performance of the cost model developed in this dissertation. In the example, inferences made in the theoretical sensitivity analysis are verified. As stated earlier, data from the New Jersey Department of Transportation, Law Engineering, and Performance Related-Specifications (1993) are used to approximate the standard deviation, σ, the fixed and variable costs c_s, c_i, the cost of acceptance and the cost of rejection parameters, λ and c_r respectively. The data set from the New Jersey Department of Transportation is available in appendix C.

Table 4.1 is an output data sheet, which contains all the parameters that are used in the testing phase. The parameter λ is calculated from an approximated cost of tolerance of $180,000.00 per lot (estimated from ERES [1993a] annual life cycle cost data) This means that, the agency representative would rather reject a product that is associated with this magnitude of cost projected for repairs, maintenance, and early replacement due to poor product performance. It can also be stated as the cost at the RQL. This cost of tolerance is divided by the square of the range of tolerance (RQL-AQL) It should be noted that the AQL takes a different meaning in this model It coincides with the optimal target value Recall that the formula for calculating λ is given in chapter three (equation 3 32). A quality index range from zero to three is used in the example. Since the quality index tables provide index values for each one hundredth interval the upper limit of the range is multiply by one hundred and then the value is squared.

The evaluation process involves appraising the responses of the model to a variety of pertinent conditions consistent with the approach taken in the sensitivity analysis. Graphs of the responses to each set of conditions along with a general discussion are presented in the subsequent section.

4.5 Discussion of Results

In order to make sure that the model would perform as intended, some precautionary measures had to be taken. All modeling and optimization procedures, as well as all mathematical operations were independently checked, and the results were satisfactory The behavior of the model during simulated conditions was impressive. It has performed to the anticipated level in all aspects. Although the results presented in the tables and graphs are self sufficient to demonstrate how the model responds to different scenarios, a summary of results is provided for completeness and to enhance clarity

The objective in the first run was to compute an optimum upper or lower limit for each sample size in the list. A set of optimum plans for sample size ranging from one to twenty-four is provided for both one- and two-sided type of quality characteristics.

For example, a plan with a sample size of ten has an optimum upper or lower limit of 348.6 psi with a probability of acceptance of 0.983, and an expected total cost of inspection $217,773.00 for the one sided case. It is to be noted that for the one-sided case the optimum limit is the same as for the two-sided case, but their probability of acceptance are different, as it should be. The probability of acceptance for the one-sided case is higher and the expected total cost of inspection is lower. For the same sample size this will always be the case. Note that for these twenty four plans a standard deviation of 300 is used. It can be observed that any plan in that table with a sample size smaller than ten is more expensive and required a wider tolerance limit. Each of these twenty four plans is a local optimum plan.

To obtain a global optimum plan, a search must be conducted to find the combination of (n, U(n)) which minimizes ETC1. A number of one dimensional search techniques could be used to do this. In this example a direct search through the values generated from MathCAD was used. A U(n) value of 348.6 was given for a sample size of ten considered above. This value was fixed and a search was conducted to locate the sample size that minimized its ETC1. Table 4.2 shows the result of the process. The global optimum plan for this tolerance value is found to have a sample size of twenty five for the one sided case. Note that ETC1 values constantly decrease until the sample size is equal twenty five, then start to increase for larger sample sizes. It can be concluded that, at this lower point, a global optimum has been reached.

Figure 4.2 shows that for each local optimum plan, the plan with a lower sample size would be preferred if BTCI were the dominant factor. However, since for each local optimum plan there is an optimum set of limits, which vary as the sample size vary, agency representatives are advised to search for a global optimum as was shown above.

4.6 Parallel with Another Approach

The optimum plan approach presented in this dissertation cannot be directly compared the plans currently used by the highway industry since they are not explicitly designed for economy. However, a limited comparison can be made using the percent defective and quality index tables. Graphs of specific quality index levels were developed and the inferences drawn are clearly consistent to that of the optimum plans. It was observed that, for each range of quality levels, there is an associated desirable sample size, which could be considered optimum. For very variable product, a sample size between 6 and 8 were found to be optimum; for more consistent material large sample sizes are needed to discriminate good product from bad material. The most important fact that was observed in both approaches was that the sample size was always greater than five. Figures 4.23 through 4.28 show the result of this effort.

4.7 Optimum Sampling Plan Implementation

Once the method presented here is used in determining the optimum plan, that is, the sample size, agency representative has to exercise judgment to either accept or not to accept such plan. This is an important aspect to note because the determined optimum plan may include undesired risk levels. In such cases, the agency representative can manipulate the program to find a satisfactory balance.

The model can be set on any mathematical software, such as mathCAD, mathLAB, or on some statistical softwares, and it can be used as a program. The researcher used mathCAD for the results provided for the example.

Table 4 1 Sensitivity Analysis and Performance Data
(local optimum plans)

n	U(n)	Pa(1)	ETCI(1)	Pa(2)	ETCI(2)
[illegible]		one-tailed		two-tailed	
1	1272.9	0.999	216219	0.999	216224
2	755.9	0.999	216333	0.999	216437
3	560.8	0.999	216547	0.993	216850
4	505.1	0.994	216791	0.999	217303
5	451.7	0.991	217001	0.993	217727
6	417.3	0.989	217197	0.978	218104
7	392.5	0.987	217389	0.975	218434
8	374.4	0.999	217521	0.971	218722
9	360.1	0.994	217654	0.992	218975
10	348.6	0.983	217773	0.988	219157
11	339.2	0.992	217879	0.984	219384
12	331.4	0.991	217975	0.982	219571
13	324.7	0.990	218062	0.980	219730
14	315.0	0.979	218141	0.959	219873
15	314.1	0.979	218214	0.957	220003
16	309.5	0.978	218291	0.956	220123
17	306.0	0.979	218343	0.955	220233
18	302.9	0.977	218402	0.954	220334
19	299.9	0.977	218456	0.953	220429
20	296.9	0.976	218508	0.952	220516
21	294.4	0.979	218556	0.951	220597
22	292.2	0.975	218602	0.951	220674
23	290.0	0.975	218649	0.950	220747
24	288.2	0.975	218697	0.949	220815

Input data:

σ = 300 psi	c_i = $15.00
λ = 2	c_s = $200.00
D = 9	c_f = $300,000.00

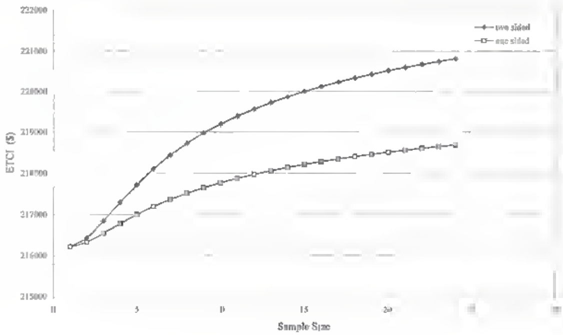


Figure 4 2 Response of the optimum sampling model to sensitivity analysis criteria
One sided and two-sided ETCI values

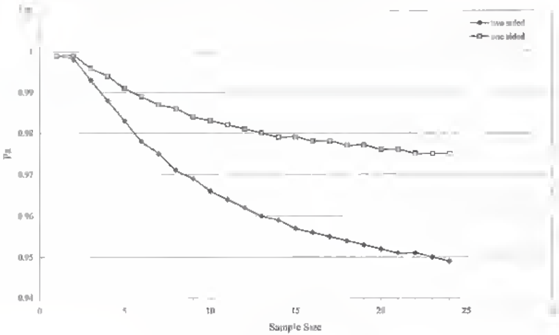


Figure 4.3 Response of the optimum sampling model to sensitivity analysis criteria
One-sided and two-sided probability of acceptance

Table 4.2 Sensitivity Analysis and Performance Data
(With fixed Limits of 348.6 psi)

[illegible]		one-tailed		two-tailed	
n	U(n)	Pa	ETCI	Pa	ETCI
1	348.6	0.956	228346	0.711	240477
2	348.6	0.918	223156	0.836	230062
3	348.6	0.944	220932	0.889	225620
4	348.6	0.958	219757	0.917	223254
5	348.6	0.967	219055	0.934	221835
6	348.6	0.972	218600	0.945	220910
7	348.6	0.976	218299	0.953	220271
8	348.6	0.979	218064	0.959	219808
9	348.6	0.981	217899	0.963	219493
10	348.6	0.983	217773	0.966	219197
11	348.6	0.984	217676	0.969	218967
12	348.6	0.985	217600	0.971	218820
13	348.6	0.986	217539	0.973	218683
14	348.6	0.987	217490	0.974	218571
15	348.6	0.988	217451	0.976	218479
16	348.6	0.988	217420	0.977	218400
17	348.6	0.989	217395	0.978	218336
18	348.6	0.989	217374	0.979	218279
19	348.6	0.99	217358	0.979	218231
20	348.6	0.99	217345	0.98	218191
21	348.6	0.99	217335	0.98	218156
22	348.6	0.99	217328	0.981	218127
23	348.6	0.991	217323	0.981	218102
24	348.6	0.991	217320	0.982	218080
25	348.6	0.991	217319	0.982	218062
26	348.6	0.991	217318	0.983	218046
27	348.6	0.991	217319	0.983	218033
28	348.6	0.992	217321	0.983	218023
29	348.6	0.992	217324	0.984	218014
30	348.6	0.992	217329	0.984	218007
31	348.6	0.992	217333	0.984	218001
32	348.6	0.992	217338	0.984	217997
33	348.6	0.992	217344	0.985	217994
34	348.6	0.992	217351	0.985	217992
35	348.6	0.992	217358	0.985	217992
36	348.6	0.993	217366	0.985	217992
37	348.6	0.993	217374	0.985	217993
38	348.6	0.993	217382	0.985	217995
39	348.6	0.993	217391	0.985	217998
40	348.6	0.993	217400	0.985	218001

σ= 500 psi	c_p= $15.00
λ= 2	c_f= $200.00
D= 5	c_r= $300 000.00

Table 4.3 Sensitivity Analysis and Performance Data
(With fixed limits –Standard Deviation = 350 psi)

		one-tailed		two-tailed	
n	U(n)	Pa(1)	ETCI(1)	Pa(2)	ETCI(2)
1	348.8	0.818	295304	0.637	296384
2	348.8	0.883	294931	0.765	295633
3	348.8	0.914	294762	0.827	295260
4	348.8	0.931	294672	0.862	295065
5	348.8	0.942	294620	0.885	294966
6	348.8	0.950	294590	0.900	294880
7	348.8	0.956	294571	0.911	294836
8	348.8	0.960	294561	0.919	294803
9	348.8	0.963	294557	0.926	294778
10	348.8	0.966	294555	0.931	294763
11	348.8	0.968	294558	0.935	294753
12	348.8	0.969	294563	0.938	294747
13	348.8	0.971	294570	0.942	294745
14	348.8	0.972	294577	0.944	294745
15	348.8	0.973	294586	0.946	294747
16	348.8	0.974	294595	0.948	294751
17	348.8	0.975	294605	0.950	294756
18	348.8	0.976	294616	0.951	294762
19	348.8	0.976	294627	0.952	294770
20	348.8	0.977	294639	0.954	294778
21	348.8	0.977	294650	0.955	294785
22	348.8	0.978	294663	0.956	294795
23	348.8	0.978	294675	0.956	294806
24	348.8	0.978	294688	0.957	294816
25	348.8	0.979	294701	0.957	294827
26	348.8	0.979	294714	0.958	294838
27	348.8	0.980	294727	0.959	294849
28	348.8	0.980	294740	0.960	294861
29	348.8	0.980	294754	0.960	294873
30	348.8	0.980	294767	0.961	294886
31	348.8	0.981	294781	0.961	294897
32	348.8	0.981	294795	0.962	294910
33	348.8	0.981	294808	0.962	294922
34	348.8	0.981	294822	0.962	294935
35	348.8	0.981	294836	0.963	294948
36	348.8	0.982	294850	0.963	294961
37	348.8	0.982	294864	0.963	294974
38	348.8	0.982	294878	0.964	294987
39	348.8	0.982	294893	0.964	295001
40	348.8	0.982	294907	0.964	295014

σ= 350 psi

λ= 2

D= 5

c_1= $15.00

c_2= $200.00

c_r= $300,000.00

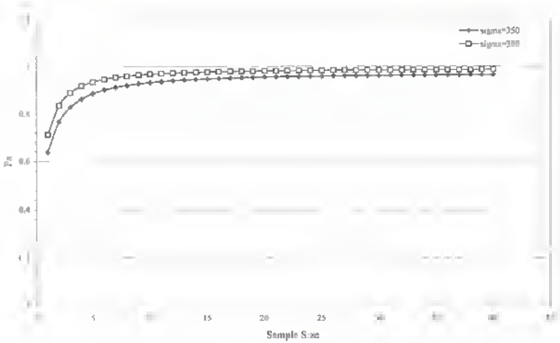


Figure 4.4 Response of the optimum sampling model to sensitivity analysis criteria
Probability of acceptance at U(n)=348.6 when σ=300 and σ=350

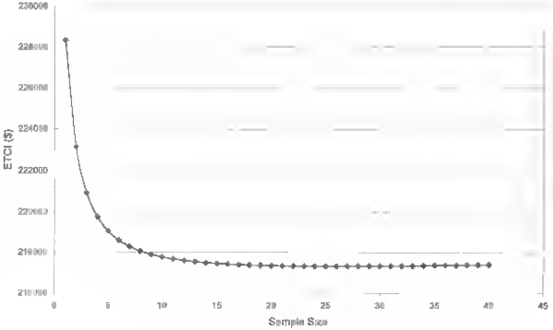


Figure 4.5 Response of the optimum sampling model to sensitivity analysis criteria ETCI values at $\sigma=300$ and U(n)=348.6

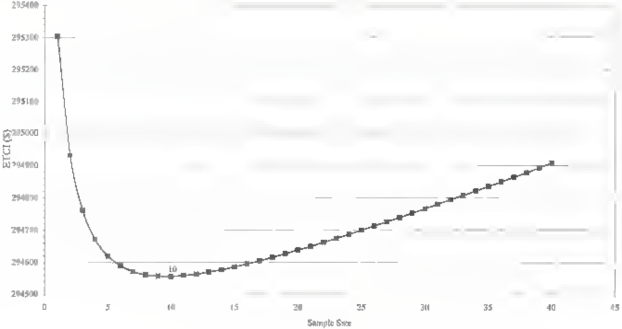


Figure 4 6 Response of the optimum sampling model to sensitivity analysis criteria ETCI values at σ=350 and U(u)=348 6

Table 4 4 Sensitivity Analysis and Performance Data
(With different costs of Rejection)

	c_r= $300 000.00			c_r= $400 000.00		
n	U(n)	Pa	ETCI	U(n)	Pa	ETCI
1	1272.8	0.999	216219	1948.3	0.999	216215
2	750.9	0.998	216333	1090.8	0.999	216231
3	585.9	0.996	216547	838.0	0.999	216257
4	500.1	0.994	216781	711.6	0.999	216287
5	451.7	0.991	217001	635.6	0.999	216340
6	417.3	0.989	217197	585.0	0.999	216407
7	392.8	0.987	217359	548.9	0.999	216469
8	374.4	0.986	217521	521.7	0.999	216530
9	360.1	0.984	217654	500.8	0.998	216590
10	348.8	0.983	217773	483.7	0.998	216648
11	339.2	0.982	217870	469.8	0.998	216703
12	331.4	0.981	217975	458.4	0.998	216756
13	324.7	0.980	218062	448.7	0.998	216807
14	318.0	0.979	218141	440.3	0.998	216855
15	314.1	0.978	218214	433.1	0.997	216901
16	308.8	0.978	218281	426.7	0.997	216945
17	305.0	0.978	218343	421.1	0.997	216987
18	302.8	0.977	218402	416.2	0.997	217027
19	299.6	0.977	218456	411.7	0.997	217066
20	296.9	0.976	218508	407.6	0.997	217103
21	294.4	0.976	218558	404.1	0.997	217139
22	292.2	0.975	218602	400.8	0.996	217174
23	290.0	0.975	218646	397.6	0.996	217207
24	288.2	0.974	218687	394.1	0.996	217239

Input data.

σ= 300 psi	c_i= $15.00
λ= 2	c_s= $200.00
D= 6	

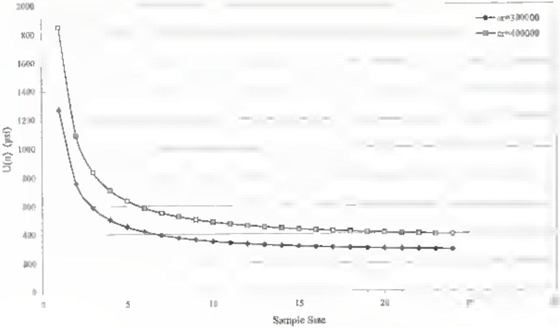


Figure 4 7 Response of the optimum sampling model to sensitivity analysis criteria U(n) values at Cr=300000 and Cr=400000

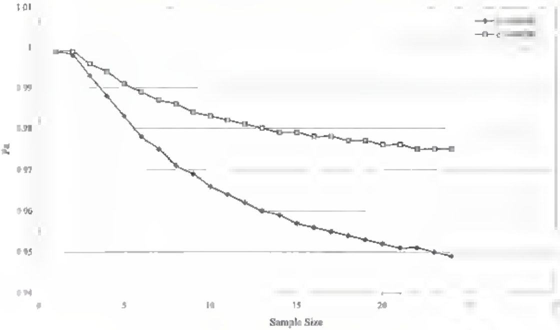


Figure 4 8 Response of the optimum sampling model to sensitivity analysis criteria
Probability of acceptance at Cr=300000 and Cr=400000

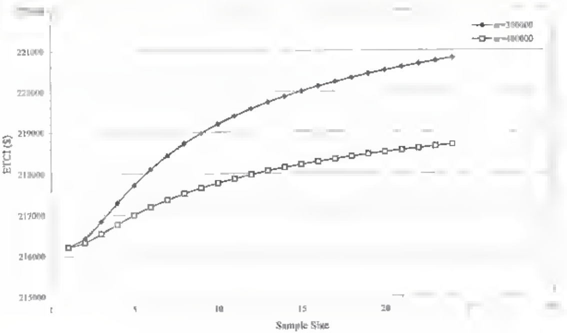


Figure4 9 Response of the opimum sampling model to sensitivity analysis criteria ETCI values at Cr=300000 and Cr=400000

Table 4 5 Sensitivity Analysis and Performance Data
(With different values of D)

	D=4			D=5		
n	U(n)	Pa	ETCI	U(n)	Pa	ETCI
1	1024.7	0.999	225299	1272.9	0.999	216219
2	636.4	0.993	225756	766.9	0.999	216333
3	508.8	0.989	226258	599.2	0.996	216547
4	441.8	0.981	226661	505.1	0.994	216781
5	402.5	0.977	226981	451.7	0.991	217001
6	378.4	0.974	227237	417.3	0.989	217197
7	357.7	0.971	227447	392.9	0.987	217369
8	343.7	0.969	227621	374.4	0.986	217521
9	332.8	0.968	227768	360.1	0.984	217654
10	324.0	0.966	227895	349.6	0.983	217773
11	318.9	0.965	228006	339.2	0.982	217879
12	310.9	0.964	228104	331.4	0.981	217979
13	306.9	0.963	228191	324.7	0.980	218062
14	301.5	0.962	228269	319.0	0.979	218141
15	297.9	0.961	228340	314.1	0.979	218214
16	294.5	0.960	228406	309.9	0.979	218281
17	291.5	0.960	228466	306.0	0.979	218343
18	289.0	0.959	228521	302.6	0.977	218402
19	286.7	0.959	228573	299.9	0.977	218456
20	284.6	0.958	228622	296.9	0.976	218509
21	282.7	0.959	228669	294.4	0.976	218558
22	281.0	0.958	228711	292.2	0.975	218602
23	279.4	0.957	228752	290.0	0.975	218649
24	279.0	0.957	228791	288.2	0.975	218697

Input data.

$\sigma = 300$ psi

$c_i = \$19.00$

$\lambda = 2$

$c_s = \$200.00$

$c_f = \$300\ 000.00$

Table 4.9 Sensitivity Analysis and Performance Data
(With different values of D)

	D=5			D=6		
n	U(n)	Pa	ETCI	U(n)	Pa	ETCI
1	1272.6	0.999	216216	1516.6	0.999	210215
2	769.9	0.998	216333	853.0	0.999	210224
3	599.8	0.996	216547	670.6	0.998	210315
4	505.1	0.994	216781	564.6	0.998	210419
5	451.7	0.991	217001	500.6	0.997	210537
6	417.3	0.989	217197	456.3	0.996	210656
7	392.6	0.987	217389	427.9	0.995	210771
8	374.4	0.986	217521	405.1	0.994	210879
9	360.1	0.984	217654	387.3	0.993	210978
10	348.9	0.983	217773	373.1	0.992	211071
11	338.2	0.982	217878	361.5	0.991	211158
12	331.4	0.981	217975	351.6	0.990	211235
13	324.7	0.980	218062	343.9	0.990	211309
14	319.0	0.979	218141	336.6	0.989	211377
15	314.1	0.979	218214	330.6	0.988	211441
16	308.6	0.978	218281	325.0	0.988	211501
17	306.0	0.978	218343	320.4	0.988	211557
18	302.6	0.977	218402	316.2	0.987	211610
19	298.6	0.977	218455	312.4	0.987	211660
20	296.0	0.976	218508	309.1	0.987	211706
21	294.4	0.976	218556	306.0	0.986	211753
22	292.2	0.975	218602	303.3	0.986	211797
23	290.0	0.975	218646	300.6	0.986	211839
24	288.2	0.976	218687	298.4	0.985	211876

Input data:
σ = 300 psi

λ = 2

c_f = $300 000.00

c_i = $16.00

c_s = $200.00

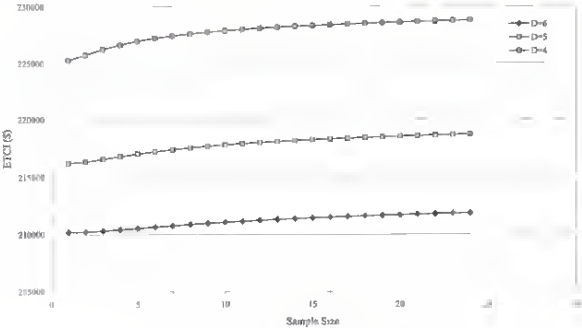


Figure 4 10 Response of the optimum sampling model to sensitivity analysis criteria ETCI values at D=4, D=5 and D=6

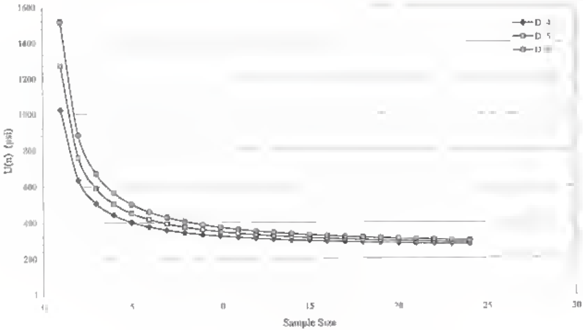


Figure 4 11 Response of the optimum sampling model to sensitivity analysis criteria Limit values at D=4, D=5 and D=6

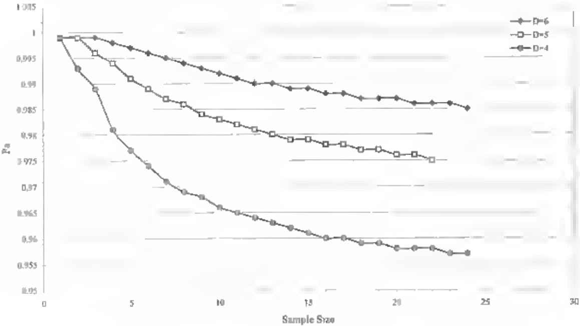


Figure 4 12 Response of the optimum sampling model to sensitivity analysis criteria
Probability of acceptance at D=4 D=5 and D=6

Table 4.7 Sensitivity Analysis and Performance Data
(With different Standard Deviations)

	σ = 300 psi			σ=350 psi		
n	U(n)	Pa	ETC	U(n)	Pa	ETCI
1	1272.8	0.999	218219	505.0	0.852	295105
2	759.0	0.999	216333	350.0	0.914	294745
3	553.8	0.996	216547	294.4	0.941	294597
4	503.1	0.994	216781	265.1	0.956	294526
5	481.7	0.991	217001	246.8	0.965	294487
6	417.3	0.989	217197	234.6	0.97	294467
7	392.9	0.987	217368	225.4	0.975	294457
8	374.4	0.986	217521	218.5	0.978	294454
9	360.1	0.984	217654	213.0	0.980	294456
10	348.8	0.983	217773	208.6	0.982	294460
11	338.2	0.982	217879	204.5	0.983	294467
12	331.4	0.981	217975	201.1	0.984	294475
13	324.7	0.980	218062	198.2	0.985	294484
14	319.0	0.979	218141	196.6	0.986	294494
15	314.1	0.978	218214	194.9	0.987	294505
16	309.9	0.978	218281	193.2	0.987	294517
17	306.0	0.978	218342	191.8	0.988	294529
18	302.6	0.977	218402	190.3	0.988	294541
19	299.0	0.977	218458	189.3	0.988	294553
20	295.9	0.976	218508	187.9	0.989	294566
21	294.4	0.976	218556	186.9	0.989	294578
22	292.2	0.975	218602	185.9	0.989	294593
23	290.0	0.975	218646	195.1	0.990	294606
24	288.2	0.975	218687	184.4	0.990	294620

Input data:

$\lambda = 2$

$D = 8$ $\quad$ $c_r = \$18.00$

$c_i = \$300\,000.00$ $\quad$ $c_p = \$200.00$

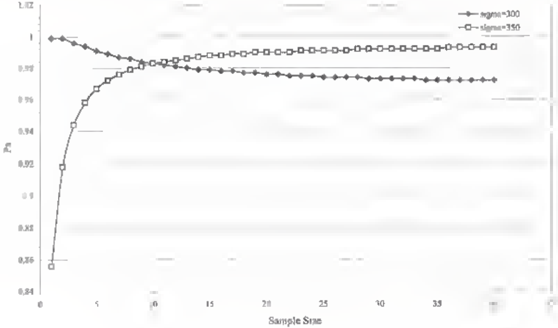


Figure4 13 Response of the optimum sampling model to sensitivity analysis criteria
Probability of acceptance values at σ=300 and σ=350

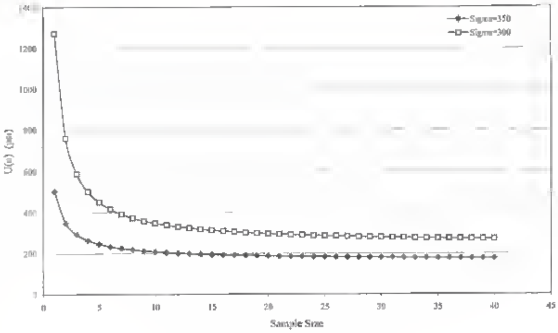


Figure 4 14 Response of the optimum sampling model to sensitivity analysis criteria Limit values at σ=300 and σ=350

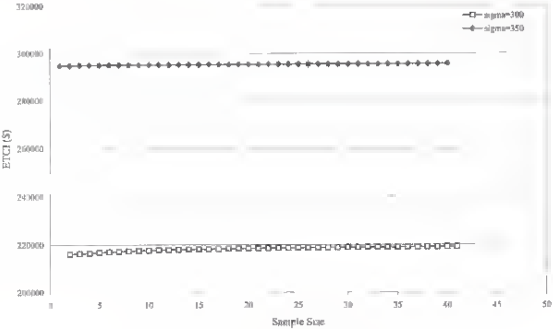


Figure 4.15 Response of the optimum sampling model to sensitivity analysis criteria ETCI values at various σ=300 and σ=350

Table 4.9 Sensitivity Analysis and Performance Data
(With fixed limits of 400 psi)

		one-tailed		two-tailed	
n	U(n)	Pa	ETCI	Pa	ETCI
1	400	0.886	225803	0.776	234892
2	400	0.944	220982	0.889	225559
3	400	0.966	219099	0.932	221947
4	400	0.977	218227	0.953	220185
5	400	0.982	217746	0.965	219216
6	400	0.988	217452	0.972	218614
7	400	0.989	217291	0.977	218219
8	400	0.990	217132	0.981	217944
9	400	0.992	217041	0.983	217748
10	400	0.993	216976	0.985	217603
11	400	0.993	216929	0.987	217493
12	400	0.994	216894	0.988	217408
13	400	0.994	216869	0.989	217343
14	400	0.995	216850	0.990	217291
15	400	0.995	216837	0.990	217250
16	400	0.995	216828	0.991	217217
17	400	0.996	216823	0.991	217191
18	400	0.996	216820	0.992	217171
19	400	0.996	216820	0.992	217155
20	400	0.996	216821	0.992	217143
21	400	0.996	216824	0.993	217134
22	400	0.996	216828	0.993	217127
23	400	0.997	216834	0.993	217123
24	400	0.997	216840	0.993	217121
25	400	0.997	216847	0.994	217120
26	400	0.997	216855	0.994	217121
27	400	0.997	216864	0.994	217123
28	400	0.997	216873	0.994	217126
29	400	0.997	216882	0.994	217130
30	400	0.997	216892	0.994	217135
31	400	0.997	216902	0.994	217140
32	400	0.997	216913	0.994	217147
33	400	0.997	216924	0.995	217153
34	400	0.997	216935	0.995	217161
35	400	0.997	216947	0.995	217169
36	400	0.997	216959	0.995	217177
37	400	0.997	216970	0.995	217186
38	400	0.997	216982	0.996	217195
39	400	0.997	216995	0.995	217205
40	400	0.998	217007	0.995	217214

σ = 300 psi

λ = 2

D = 5

c_s = $15.00

c_f = $200.00

c_r = $300 000.00

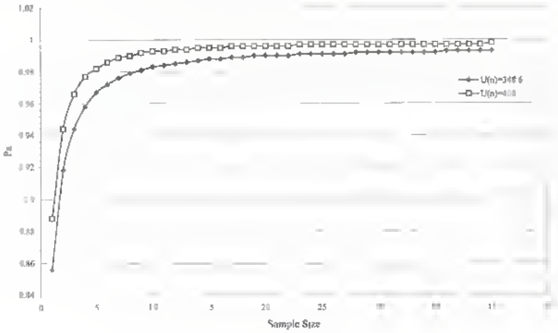


Figure 4 16 Response of the optimum sampling model to sensitivity analysis criteria
One-sided probability of acceptance at U(n)=348 6 and U(n)=400

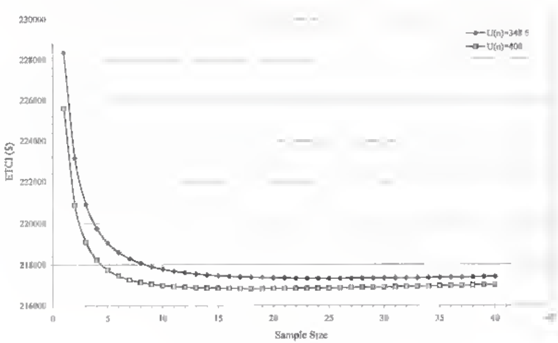


Figure417 Response of the optimum sampling model to sensitivity analysis criteria
One-sided ETCI values at U(n)=348.6 and U(n)=400

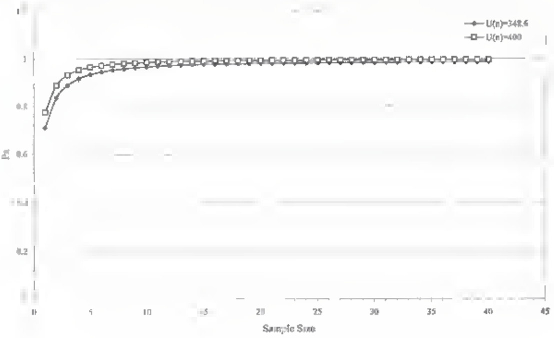


Figure 4 18 Response of the optimum sampling model to sensitivity analysis criteria
Two-sided probability of acceptance at U(n)=348.6 and U(n)=400

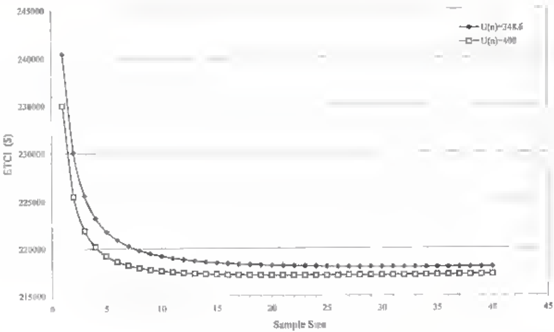


Figure 4 19 Response of the optimum sampling model to sensitivity analysis criteria
Two-sided ETCI values at U(n)=348 6 and U(n)=400

Table 4.8 Sensitivity Analysis and Performance Data
(With different costs of Acceptance)

	$\lambda = 2$			$\lambda = 1.75$		
n	U(n)2	Pa(2)	ETCI(2)	U(n)1.75	Pa(1.75)	ETCI(1.75)
1	1272.8	0.999	216219	1548.4	0.999	189215
2	750.9	0.998	216333	916.5	0.999	189244
3	565.6	0.996	216547	708.4	0.999	189315
4	305.1	0.994	216781	601.3	0.999	189415
5	451.7	0.991	217001	536.2	0.999	189527
6	417.3	0.989	217197	496.2	0.997	189640
7	392.8	0.987	217369	466.1	0.996	189747
8	374.4	0.986	217521	443.9	0.995	189847
9	360.1	0.984	217654	426.0	0.995	189939
10	348.9	0.983	217773	411.9	0.994	190025
11	339.2	0.982	217879	400.9	0.993	190104
12	331.4	0.981	217975	390.8	0.993	190177
13	324.7	0.980	218062	382.8	0.992	190245
14	319.0	0.979	218141	375.8	0.992	190309
15	314.1	0.979	218214	369.8	0.992	190367
16	309.9	0.978	218281	364.5	0.991	190422
17	306.0	0.978	218343	359.8	0.991	190474
18	302.8	0.977	218402	355.8	0.991	190523
19	299.6	0.977	218456	352.0	0.990	190570
20	296.9	0.976	218506	348.7	0.990	190614
21	294.4	0.976	218555	345.7	0.990	190656
22	292.2	0.975	218602	342.9	0.989	190696
23	290.0	0.975	218646	340.5	0.989	190735
24	288.2	0.975	218687	338.1	0.989	190772

Input data
σ = 300 psi c_i = $15.00

D = 5 c_a = $200.00

c_f = $300,000.00

Table 4 10 Sensitivity Analysis and Performance Data
(With different costs of Acceptance)

	λ= 2			λ= 2 25		
n	U(n)2	Pa(2)	ETCI(2)	U(n)2 25	Pa(2 25)	ETCI(2 25)
1	1272.8	0.999	216219	1009.8	0.998	243275
2	759.8	0.998	216333	611.0	0.993	243655
3	586.8	0.996	216547	477.8	0.985	244078
4	500.1	0.994	216781	410.8	0.978	244434
5	451.7	0.991	217001	370.8	0.975	244722
6	417.3	0.989	217197	343.7	0.971	244956
7	392.5	0.987	217399	324.5	0.968	245145
8	374.4	0.986	217521	310.1	0.965	245310
9	360.1	0.984	217654	298.8	0.963	245447
10	348.8	0.983	217773	289.8	0.961	245568
11	339.2	0.982	217879	282.4	0.960	245670
12	331.4	0.981	217975	276.3	0.958	245762
13	324.7	0.980	218082	271.1	0.957	245844
14	318.0	0.979	218141	266.9	0.956	245918
15	314.1	0.978	218214	262.7	0.955	245986
16	309.8	0.978	218281	259.3	0.954	246048
17	306.0	0.978	218343	256.4	0.953	246106
18	302.8	0.977	218402	253.7	0.953	246159
19	299.8	0.977	218456	251.3	0.952	246208
20	296.8	0.976	218503	249.1	0.952	246255
21	294.4	0.976	218556	247.2	0.951	246298
22	292.2	0.975	218602	245.4	0.951	246341
23	290.0	0.975	218646	243.5	0.950	246381
24	288.2	0.975	218687	242.4	0.950	246418

Input data.

σ= 300 psi	c_i= $18.00
D= 5	c_a= $200.00
c_f= $300 000.00	

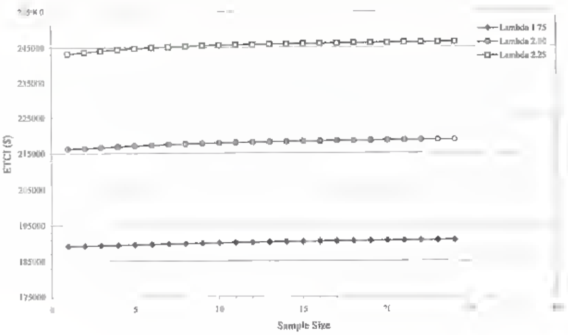


Figure 4.20 Response of the optimum sampling model to sensitivity analysis criteria ETCI values at λ=1.75, λ=2.00, and λ=2.25

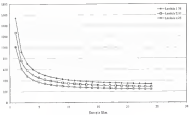


Figure 4.21 Response of the optimum sampling model to sensitivity analysis criteria
Limit values for at $\lambda=1.75$, $\lambda=2.00$, $\lambda=2.25$

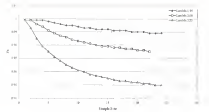


Figure4.22 Response of the optimum sampling model to sensitivity analysis criteria
Probability of acceptance at $\lambda=1.75$, $\lambda=2.00$ and $\lambda=2.25$

Table 4 11 Sensitivity Analysis and Performance Data
(With different fixed costs of Inspection)

	c_f= $200.00			c_f= $400.00		
n	U(n)2	Pa	ETCI	U(n)	Pa	ETCI
1	1272.8	0.999	216218	1272.8	0.999	216418
2	759.9	0.990	216333	759.8	0.999	216533
3	568.8	0.995	216647	568.9	0.996	216747
4	305.1	0.994	216781	305.1	0.994	216981
5	451.7	0.991	217001	451.7	0.991	217201
6	417.3	0.989	217197	417.3	0.989	217397
7	392.8	0.987	217359	392.8	0.987	217569
8	374.4	0.986	217521	374.4	0.986	217721
9	360.1	0.984	217694	360.1	0.984	217894
10	348.6	0.983	217773	348.6	0.983	217873
11	339.2	0.982	217878	339.2	0.982	218179
12	331.4	0.981	217978	331.4	0.981	218175
13	324.7	0.980	218062	324.7	0.980	218262
14	318.0	0.979	218141	318.0	0.979	218341
15	314.1	0.979	218214	314.1	0.979	218414
16	309.8	0.978	218281	308.8	0.978	218461
17	306.0	0.978	218343	306.0	0.978	218543
18	302.6	0.977	218402	302.8	0.977	218602
19	298.8	0.977	218459	298.6	0.977	218656
20	296.9	0.976	218508	296.8	0.976	218708
21	294.4	0.976	218556	294.4	0.976	218756
22	292.2	0.975	218602	292.2	0.976	218802
23	290.0	0.976	218646	290.0	0.975	218846
24	288.2	0.976	219687	288.2	0.975	218887

Input data
σ= 300 psi c= $15.00
D= 8 λ= 2
c_f= $300 000.00

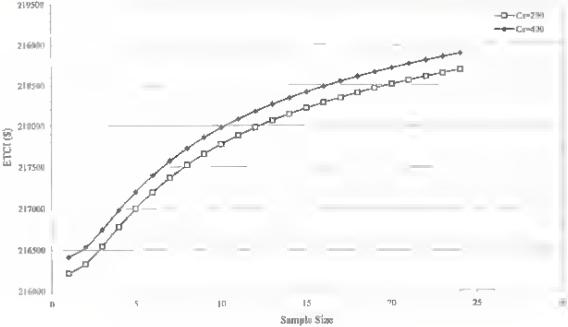


Figure 4 23 Response of the optimum sampling model to sensitivity analysis criteria ETCI values at Cs=200 and Cs=400

Table 4.12 Estimation of lot percent defective based on quality index and sample size
(Modified from FHWA's performance-related specifications for concrete pavements)

Quality Index (Q)													sample size
	3	4	5	6	7	8	9	10	15	20	30	50	100
0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
0.01	49.72	49.67	49.64	49.63	49.63	49.62	49.62	49.62	49.61	49.61	49.60	49.60	49.60
0.02	49.45	49.33	49.29	49.27	49.25	49.24	49.24	49.23	49.22	49.21	49.21	49.21	49.20
0.03	49.17	49.00	48.93	48.90	48.88	48.86	48.85	48.85	48.83	48.82	48.81	48.81	48.81
0.04	48.90	48.67	48.58	48.53	48.50	48.48	48.47	48.46	48.44	48.43	48.42	48.41	48.41
0.05	48.62	48.33	48.22	48.16	48.13	48.11	48.09	48.08	48.05	48.04	48.02	48.02	48.01
0.06	48.35	48.00	47.86	47.80	47.76	47.73	47.71	47.70	47.66	47.64	47.63	47.62	47.61
0.07	48.07	47.67	47.51	47.43	47.38	47.35	47.33	47.31	47.27	47.25	47.24	47.22	47.22
0.08	47.79	47.33	47.15	47.06	47.01	46.97	46.95	46.93	46.88	46.86	46.84	46.83	46.82
0.09	47.52	47.00	46.80	46.70	46.63	46.59	46.57	46.54	46.49	46.47	46.45	46.43	46.42
0.1	47.24	46.67	46.44	46.33	46.26	46.22	46.18	46.16	46.10	46.08	46.05	46.04	46.03
0.11	46.96	46.33	46.09	45.96	45.89	45.84	45.80	45.78	45.71	45.69	45.66	45.64	45.63
0.12	46.69	46.00	45.73	45.60	45.51	45.46	45.42	45.40	45.33	45.29	45.27	45.25	45.24
0.13	46.41	45.67	45.38	45.23	45.14	45.08	45.04	45.01	44.94	44.90	44.88	44.86	44.84
0.14	46.13	45.33	45.02	44.86	44.77	44.71	44.66	44.63	44.55	44.51	44.48	44.46	44.45
0.15	45.85	45.00	44.67	44.50	44.40	44.33	44.29	44.25	44.16	44.13	44.09	44.07	44.05

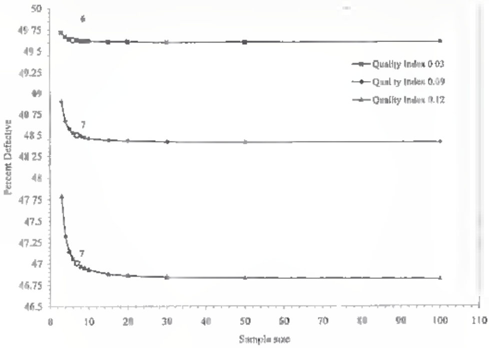


Figure 4.24 Optimal sample size approximation using percent defective and quality index

Table 4.13 Estimation of lot percent defective based on quality index and sample size
(Modified from FHA's performance related specifications for concrete pavements)

Quality index (Q)	3	4	5	6	7	8	9	10	15	20	30	50	sample size 100
0.37	38.62	37.67	36.93	36.55	36.33	36.19	36.08	36.02	35.83	35.75	35.69	35.63	35.[illegible]
0.38	38.33	37.33	36.58	36.20	35.98	35.83	35.73	35.65	35.48	35.38	35.31	35.26	35.23
0.39	38.03	37.00	36.23	35.84	35.62	35.47	35.37	35.29	35.10	35.01	34.94	34.89	34.86
0.40	36.74	36.67	35.88	35.49	35.26	35.11	35.00	34.93	34.73	34.65	34.58	34.52	34.49
0.41	38.15	36.33	35.54	35.14	34.90	34.75	34.64	34.57	34.37	34.28	34.21	34.16	34.12
0.42	36.15	36.00	35.19	34.79	34.[illegible]	34.39	34.29	34.21	34.00	33.92	33.85	33.79	33.76
0.43	37.85	35.67	34.85	34.43	34.19	34.04	33.93	33.[illegible]	33.64	33.56	33.48	33.43	33.39
0.44	37.06	35.33	34.50	34.06	33.84	33.69	33.57	33.40	33.28	33.20	33.12	33.07	33.03
0.45	37.28	35.00	34.16	33.73	33.49	33.33	33.21	33.13	32.92	32.84	32.76	32.71	32.69
0.46	36.90	34.67	33.81	33.38	33.13	32.97	32.86	32.78	32.57	32.49	32.40	32.35	32.31
0.47	36.66	34.33	33.47	33.04	32.78	32.62	32.51	32.42	32.21	32.12	32.04	31.99	31.95
0.48	36.35	34.00	33.12	32.69	32.43	32.27	32.16	32.07	31.85	31.77	31.69	31.63	31.60
0.49	36.05	33.67	32.78	32.34	32.08	31.92	31.80	31.72	31.50	31.41	31.33	31.28	31.24
0.50	35.75	33.33	32.44	32.00	31.74	31.57	31.45	31.37	31.15	31.06	30.98	30.93	30.89
0.51	35.44	33.00	32.10	31.65	31.39	31.22	31.10	31.02	30.80	30.71	30.63	30.57	30.54

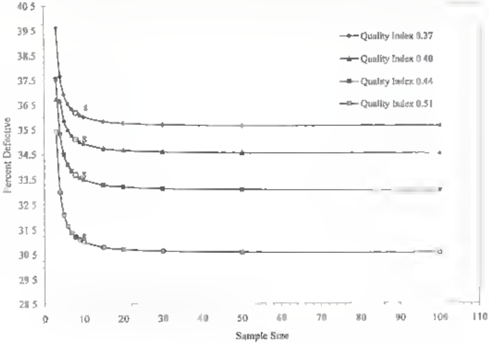


Figure 4.25 Optimal sample size approximation using percent defective and quality index

Table 4.14 Estimation of lot percent defective based on quality index and sample size
(Modified from FHA's performance-related specifications for concrete pavements)

Quality index (Q)													sample size
	3	4	5	6	7	8	9	10	15	20	30	50	100
1.00	16.67	16.67	16.36	16.20	16.1	16.04	16.00	15.97	15.91	15.89	15.88	15.87	15.87
1.01	16.11	16.33	16.07	15.92	15.83	15.76	15.74	15.72	15.66	15.64	15.63	15.63	15.62
1.02	15.53	16.00	15.76	15.64	15.56	15.51	15.48	15.45	15.41	15.40	15.39	15.38	15.38
1.03	14.93	15.67	15.48	15.37	15.30	15.25	15.23	15.21	15.17	15.15	15.15	15.16	15.15
1.04	14.31	15.33	15.16	15.08	15.03	15.00	14.97	14.96	14.92	14.91	14.91	14.91	14.91
1.05	13.66	15.00	14.81	14.82	14.77	14.74	14.72	14.71	14.68	14.67	14.67	14.66	14.66
1.06	12.98	14.67	14.62	14.55	14.51	14.49	14.47	14.46	14.44	14.44	14.44	14.45	14.45
1.07	12.27	14.33	14.33	14.26	14.26	14.24	14.21	14.22	14.2	14.2	14.21	14.22	14.22
1.08	11.51	14.00	14.05	14.02	14.00	13.99	13.98	13.91	13.97	13.67	13.98	13.99	14.00
1.09	10.71	13.67	13.76	13.76	13.75	13.74	13.73	13.73	13.74	13.74	13.75	13.77	13.77
1.10	9.84	13.33	13.46	13.50	13.46	13.49	13.46	13.50	13.61	13.52	13.53	13.54	13.55
1.11	8.89	13.00	13.20	13.24	13.26	13.25	13.26	13.26	13.28	13.29	13.31	13.32	13.34
1.12	7.82	12.67	12.93	12.96	13.00	13.01	13.02	13.03	13.06	13.07	13.09	13.11	13.12
1.13	6.60	12.33	12.56	12.72	12.76	12.77	12.79	12.80	12.83	12.85	12.87	12.89	12.91
1.14	5.08	12.00	12.37	12.47	12.51	12.54	12.55	12.57	12.61	12.63	12.66	12.68	12.7
1.15	2.87	11.67	12.10	12.22	12.27	12.30	12.32	12.34	12.39	12.42	12.45	12.47	12.49

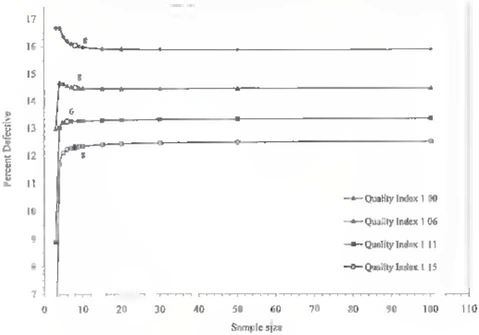


Figure 4.26 Optimal sample size approximation using percent defective and quality index

Table 4.15 Estimation of lot percent defective based on quality index and sample size
(Modified from FHA's performance-related specifications for concrete pavements)

Quality index (Q)	3	4	5	6	7	8	9	10	15	20	30	50	sample size 100
1.53	0.00	0.00	3.23	4.31	4.82	5.12	5.31	5.45	5.80	5.94	6.07	6.17	6.24
1.54	0.00	0.00	3.05	4.18	4.67	4.97	5.17	5.31	5.67	5.81	5.95	6.04	6.11
1.55	0.00	0.00	2.87	4.00	4.52	4.83	5.03	5.18	5.54	5.69	5.82	5.92	5.99
1.56	0.00	0.00	2.69	3.84	4.38	4.69	4.90	5.05	5.41	5.56	5.70	5.80	5.87
1.57	0.00	0.00	2.52	3.69	4.24	4.56	4.77	4.92	5.29	5.44	5.58	5.68	5.75
1.58	0.00	0.00	2.35	3.54	4.1	4.42	4.64	4.79	5.16	5.32	5.46	5.56	5.64
1.59	0.00	0.00	2.19	3.40	3.96	4.25	4.51	4.66	5.04	5.20	5.34	5.45	5.52
1.60	0.00	0.00	2.03	3.25	3.83	4.16	4.38	4.54	4.92	5.08	5.23	5.33	5.41
1.61	0.00	0.00	1.87	3.11	3.69	4.03	4.26	4.41	4.81	4.97	5.12	5.22	5.30
1.62	0.00	0.00	1.72	2.97	3.57	3.91	4.14	4.3	4.69	4.86	5.01	5.11	5.19
1.63	0.00	0.00	1.57	2.84	5.44	3.79	4.02	4.18	4.58	4.75	4.90	5.01	5.08
1.64	0.00	0.00	1.42	2.71	3.31	3.67	3.90	4.06	4.47	4.64	4.79	4.90	4.98
1.65	0.00	0.00	1.28	2.58	3.18	3.35	3.78	3.95	4.36	4.53	4.68	4.78	4.87
1.66	0.00	0.00	1.15	2.45	3.07	3.43	3.67	3.84	4.25	4.43	4.58	4.68	4.77
1.67	0.00	0.00	1.02	2.33	2.96	3.32	3.56	3.73	4.15	4.32	4.48	4.59	4.67
1.68	0.00	0.00	0.89	2.21	2.84	3.21	3.45	3.62	4.05	4.22	4.38	4.49	4.57

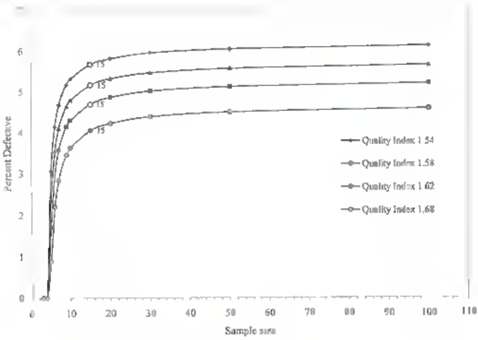


Figure 4.27 Optimal sample size approximation using percent defective and quality index

Table 4.16 Estimation of lot percent defective based on quality index and sample size
(Modified from FHA's performance related specifications for concrete pavements)

Quality Index (Q)	3	4	5	6	7	8	9	10	15	20	30	50	sample size 100
2.06	0.00	0.00	0.00	0.00	0.23	0.52	0.73	0.90	1.33	1.51	1.67	1.80	1.89
2.07	0.00	0.00	0.00	0.00	0.21	0.48	0.7	0.86	1.28	1.47	1.63	1.75	1.84
2.08	0.00	0.00	0.00	0.00	0.18	0.45	0.68	0.82	1.24	1.42	1.58	1.71	1.79
2.09	0.00	0.00	0.00	0.00	0.16	0.42	0.62	0.78	1.20	1.38	1.54	1.66	1.75
2.1	0.00	0.00	0.00	0.00	0.14	0.39	0.58	0.74	1.16	1.34	1.50	1.62	1.71
2.11	0.00	0.00	0.00	0.00	0.12	0.36	0.55	0.71	1.12	1.30	1.46	1.58	1.66
2.12	0.00	0.00	0.00	0.00	0.10	0.33	0.52	0.67	1.08	1.26	1.42	1.54	1.62
2.13	0.00	0.00	0.00	0.00	0.09	0.30	0.49	0.64	1.04	1.22	1.38	1.50	1.58
2.14	0.00	0.00	0.00	0.00	0.07	0.28	0.46	0.61	1.00	1.18	1.34	1.46	1.54
2.15	0.00	0.00	0.00	0.00	0.06	0.26	0.43	0.58	0.97	1.14	1.30	1.42	1.50
2.16	0.00	0.00	0.00	0.00	0.05	0.23	0.41	0.55	0.93	1.10	1.26	1.38	1.46
2.17	0.00	0.00	0.00	0.00	0.04	0.21	0.39	0.52	0.90	1.07	1.22	1.34	1.42
2.18	0.00	0.00	0.00	0.00	0.03	0.19	0.36	0.49	0.87	1.03	1.19	1.30	1.39
2.19	0.00	0.00	0.00	0.00	0.02	0.17	0.33	0.46	0.83	1.00	1.15	1.27	1.35
2.2	0.00	0.00	0.00	0.00	0.01	0.16	0.31	0.44	0.80	0.97	1.12	1.23	1.31
2.21	0.00	0.00	0.00	0.00	0.01	0.14	0.29	0.41	0.77	0.94	1.09	1.20	1.28

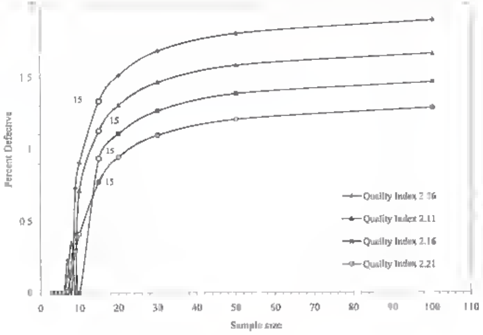


Figure 4.28 Optimal sample size approximation using percent defective and quality index

Table 4.17 Estimation of lot percent defective based on quality index and sample size
(Modified from FHA's performance-related specifications for concrete pavements)

Quality index (Q)	3	4	5	6	7	8	9	10	15	20	30	50	sample size 100
2.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.13	0.21	0.29	0.36	0.41
2.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.12	0.2	0.28	0.35	0.40
2.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.11	0.19	0.27	0.34	0.39
2.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.11	0.18	0.26	0.33	0.38
2.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.10	0.17	0.25	0.32	0.37
2.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.16	0.24	0.31	0.35
2.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.16	0.23	0.30	0.34
2.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.15	0.22	0.29	0.33
2.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.14	0.22	0.28	0.32
2.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.14	0.21	0.27	0.31
2.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.13	0.20	0.26	[illegible]
2.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.12	0.19	0.25	0.29
2.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.12	0.18	0.24	0.28
2.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.11	0.18	0.23	0.27
2.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.11	0.17	0.22	0.27
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10	0.16	0.22	0.26

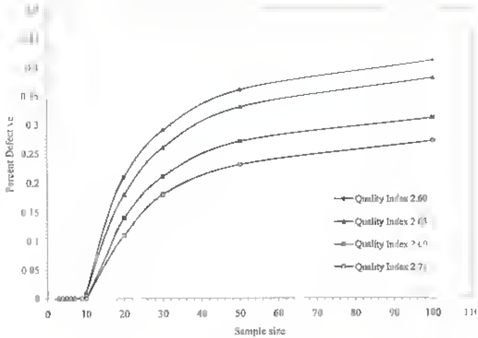


Figure 4.25 Optimal sample size approximation using percent defective and quality index

Table 4.18 Estimation of lot percent defective based on quality index and sample size
(Modified from FHA's Performance-Related Specifications for Concrete Pavements)

Quality index (Q)	3	4	5	6	7	8	9	10	15	20	30	50	sample size 100
2.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.08	0.13	0.18	0.22
2.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.13	0.17	0.21
2.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.12	0.17	0.20
2.83	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.12	0.16	0.20
2.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.06	0.11	0.16	0.19
2.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.11	0.15	0.18
2.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.10	0.14	0.18
2.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.10	0.14	0.17
2.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.09	0.13	0.17
2.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.09	0.13	0.16
2.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.09	0.12	0.16
2.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.08	0.12	0.15
2.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.08	0.12	0.14
2.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.08	0.11	0.14
2.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.07	0.11	0.13
2.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.10	0.13

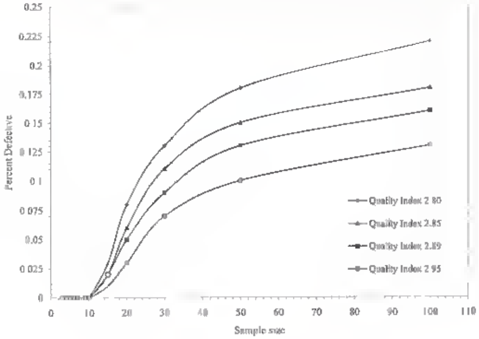


Figure 4 30 Optimal sample size approximation using percent defective and quality index

CHAPTER 5
PAY ADJUSTMENT

In chapter one, the researcher mentioned the enactment of the Intermodal Surface Transportation Efficiency Act (ISTEA) in 1993. Its requirements include the consideration of the life cycle cost of highway products in the decision making process of the transportation agency, not the first cost of products only. Also in chapter one, in the statement of objectives, the researcher indicated that the consequences of decisions made in acceptance procedures have to be addressed in terms of the risks involved and the resulting pay-adjustment to the contractor. This chapter deals with the necessary formulation for life cycle cost analyses and the adjusting pay factor.

ERES [1993a, 1993b] prepared a report, as part of a government research project, on Performance Related Specifications (PRS). The researcher finds the PRS report very innovative and conceptually consistent with this research in the large sense and concludes that this research will complement the PRS effort in many areas. However, a different approach is taken in this dissertation in formulating a pay-adjustment procedure using the life cycle costs of Portland cement pavement.

5.1 Pavement Life Cycle Cost PRS Formulation

The researcher finds it necessary to point out once again the main objective of this research, which was to develop a methodology to determine the optimum sample size. As was evident in the last chapter, this could only have been done through cost consideration. This effort would have been futile if only the sample size was optimized. The researcher intends to revisit this subject later in this chapter.

ERES consultants [1993a, 1993b] in their report on Performance-Related Specification suggest the application of the life cycle cost of pavement as the overall quality measure. To do this, it must be estimated for both as-designed and the as-constructed pavements. On this estimation rests the validity of PRS. Life cycle cost in the specification consists of the estimated future rehabilitation costs, user costs, and salvage value at the end of the analysis period, and is expressed in terms of present worth through the use of a specified discount rate. The initial construction cost is not included since it is identical to both the as-designed and the as-constructed pavements.

The pay factor is calculated by considering the difference between the target as-designed life cycle costs of pavement and the as-constructed pavement life cycle costs in conjunction with the bid price. A lower lot life cycle cost indicates an increase in quality while a higher lot life cycle cost indicates a decrease in quality.

According to ERES [1993c] in the pavement design process, the designer selects various inputs to develop the pavement design. When using the 1986 AASHTO guide, mean values of all input are required by the procedure, including strength, slab support, traffic, initial serviceability, and so on. The guide also requires an overall standard deviation, which consists of an estimate of the uncertainty in the prediction of future traffic loading and the error associated with the prediction of performance. The variations of quality characteristics, such as slab thickness, concrete strength, and initial serviceability are included in the estimation of an overall standard deviation. The design output is the mean slab thickness required for a selected level of design reliability and is obtained during construction. The mean concrete strength and initial serviceability are expected to be obtained during construction to achieve the given level of design reliability. Thus, when using the AASHTO guide for pavement design, the mean values used for inputs and the output thickness are specified in PRS as the mean quality characteristics for the as-designed pavement. They are specified as target values to be achieved by the contractor in order to receive full pay.

The pay factor proposed in PRS is expressed as a percentage of the bid price that the contractor is paid for the construction of a lot of concrete pavement. It is calculated as follows:

$$PayFactor = 100\frac{(BID + DIFF)}{BID} \tag{5.1}$$

where

BID = Contractor's bid price for the lot, $

DIFF = $LCC_{des} - LCC_{con}$

LCC_{des} = As designed life cycle cost for the lot, $

LCC_{con} = As constructed lifecycle cost for the lot, $

PRS also considers a percent defective approach to pay adjustment. It is defined as the percentage of the lot falling above the mean target as-designed life cycle cost value. The life cycle cost for the as-designed pavement is computed through simulation using the target values for strength, thickness, air content, and roughness (see Figure 5.1). The as-designed life cycle is estimated by simulating at least one hundred lots. The agency uses this as-designed life cycle target value to compare with the life cycle computed from the as-constructed lot life cycle cost (see Figure 5.2) The percent defective of the as-constructed lot is defined as the proportion of the lot having a life cycle as-constructed greater than the as-designed The percent defective Q, is calculated as follows

$$Q = \frac{(LCC_{des} - LCC_{con})}{S_{con}} \tag{5.2}$$

Here S_{con} is the standard deviation of life cycle cost between as-constructed sublots in dollars. The contractor is paid based upon the achieved quality of the as-constructed pavement lot The payment to the contractor is adjusted when the constructed pavement lot quality level varies from the as-designed pavement quality level. When a contractor constructs a lot that has a life cycle cost that exactly equals the as-designed life cycle, there is no need for a pay adjustment and the contractor receive full pay for the pavement. When there is a difference between these values, the contractor bid will be adjusted according to the difference That is, if as-constructed life cycle cost is less than as-designed life cycle cost there is a positive adjustment. Conversely if as-constructed life cycle cost is greater than as-designed life cycle cost there is a negative adjustment

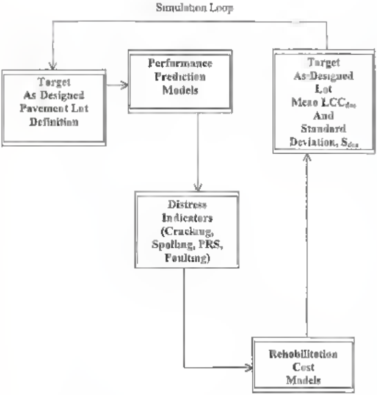

Simulation Loop
Target As Designed Pavement Lot Definition
Performance Prediction Models
Target As-Designed Lot Mean LCC_{des} And Standard Deviation, S_{des}
Distress Indicators (Cracking, Spalling, PRS, Faulting)
Rehabilitation Cost Models

Figure 5 1 PRS procedures for estimation of as-designed life cycle (mean LCC_{des} from simulation with target S T A, and R)

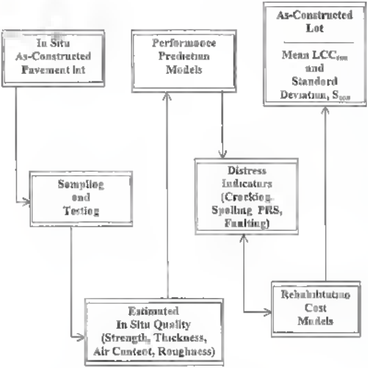

In Situ
As-Constructed
Pavement lot
Performance
Prediction
Models
As-Constructed
Lot
Mean LCC_{con}
and
Standard
Deviation, S_{con}
Sampling
and
Testing
Distress
Indicators
(Cracking,
Spalling, PRS,
Faulting)
Estimated
In Situ Quality
(Strength, Thickness,
Air Content, Roughness)
Rehabilitation
Cost
Models

Figure 5.2 PRS procedures for estimation of as-constructed life cycle

alternative as their choice. However, there are certain situations where one method is more appropriate than another. In the case of pavement life cycle, two of the three methods could be used where appropriate. That is, annual worth method, and the present worth method, with the preference being the annual worth method. The reason for this is simple. When calculating life cycle cost to compare mutually exclusive alternatives with possibly different lives using the present worth method, in addition to very laborious computation, the solution, in terms of present worth, offers very little insight and can be outright misleading. For example, a table in ERES' PRS, which shows the prediction of a future rehabilitation cost for an as-constructed sublot. The cost data is spread over the forty year life cycle. But what is not so clear, is the fact that these costs are expressed in terms of Present Worth. That is, they have exactly the same value as today's money. A more physical meaning is obtained from the annual, where the analysis period becomes less significant. For budgetary reasons, an agency might want to see alternatives expressed in terms of their annual worth since cost of rehabilitation and maintenance expenditures are allocated on an annual basis. This is really not a major concern. However, a change would avoid misinterpretations. The matter was addressed here in the interest of completeness. A much more important concern regarding pay adjustment will be addressed in the next sections.

5.2 Basis for Target Value

In Performance Related Specifications, ERES indicated that the life cycle cost is not used to compare alternative designs. There lies the fundamental difference between the ERES approach and the researcher's approach. Even though life cycle cost analysis is stated as the corner stone of the method, its application is too limiting. Earlier in this research, it was shown that design of pavement should be based on target value and that standard deviation should be used to control variability in the system. It is important to remember that a target value and standard deviation are not sufficient to achieve the quality desired. That is, if one needs to use life cycle as an overall quality measure, it must be used differently from the manner in which it is used in performance-related specifications. In addition to the target value and the standard deviation, the researcher proposed that the optimum value be used since optimum has been defined as the best design for the lowest life cycle cost.

This is achieved through parameter design discussed in Appendix B. This missing step in current design process is crucial to achieve optimum quality. During the design process, as it is explained in more detail in the next chapter, control levels that render the design insensitive to noise are sought and the life cycle cost determined for the many possibilities. The result is a robust and optimal pavement design. It is to be noted that with this approach, that the as-designed pavement will always be superior to the as-constructed pavement. Consider an example given in performance related specifications [p61]. The mean of an as-constructed life cycle cost is $41,162.00/km. So the percent defective was calculated and yielded the following result.

$$Q = \frac{(LCC_{des} - LCC_{con})}{S_{des}} = \frac{\$43{,}288 - \$41{,}162}{\$3146} = +0.676$$

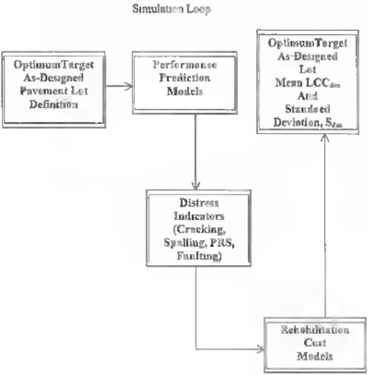
The point that needs to be made in this example is that the contractor was able to come up with a product that is of higher quality and is also cheaper than the as-designed pavement and surely will receive a bonus The researcher maintains that the lower cost alternative should have been discovered and selected by the design engineer through parameter design. See Figure 5 3

Another important point that needs attention here concerns the quality characteristics, in particular compressive strength This quality characteristic has more than one functional quality characteristics The manner, in which they are used in current specifications to include Performance-Related Specifications, also needs reviewing The link between the compressive strength and its functional quality characteristics has to be established.

5 3 Determination of the Constants λ'and c_r

The efficiency and effectiveness of the economic model presented in the last chapter is strongly dependent upon the accurate assessment of its cost components and cost parameters ERES computer program Pave-Spec appears to possess the capability to make reliable estimations of life cycle cost of pavement for any given analysis period through the use of distress models and economic analysis principles Again, It is to be emphasized that the attainment of this objective is crucial to optimal performance of the model

In the model presented in the last chapter two of the key factors that have to be determined are λ and c_r λ is the quality cost parameter for accepting a defective, and


Simulation Loop

OptimumTarget
As-Designed
Pavement Lot
Definition
Performance
Prediction
Models
OptimumTarget
As-Designed
Lot
Mean LCC_{des}
And
Standard
Deviation, S_{des}
Distress
Indicators
(Cracking,
Spalling, PRS,
Faulting)
Rehabilitation
Cost
Models

Figure 5 3 proposed procedures for estimation of as-designed life-cycle (mean LCC_{des} from simulation with target S T A and R)

c_r is the cost of rejecting a lot. The researcher only provides concepts and guidelines for the computation of 'λ' and c_r. It is expected that each agency will establish the proper balancing of the cost components that yield a 'λ' and c_r that best suit their situation.

The Taguchi loss function quantifies the variability in a product or process. Therefore, if quality control data show that an installed pavement's mean value does not lie within the specification limits, the pavement should be replaced. This follows the presumption that the specifications are related to the reliability of the pavement. Hence as the specification limits are approached, the pavement is less likely to perform satisfactorily. Once this state exists, other costs are incurred by the highway agency in the form of excessive cost of rehabilitation, repair and premature replacement in addition to user cost.

In quality control terminology, the cost parameter 'λ' is the ratio of the life cycle cost at the rejectable quality level (RQL) to the square of width of the specification limit. The resulting 'λ' is term of cost per square unit of variability.

5.4 Proposed Method for Calculating Pay Factor

Another useful potential application of the quadratic loss function is the area of adjusted pay factor calculation. The quadratic loss function embodies the necessary properties to efficiently adjust the contractor's pay. It has been used throughout this dissertation to account for loss in quality. And the quality loss is expressed in terms of life cycle cost; hence, it is fitting to use the same approach to determine the appropriate pay adjustment factor. The proposed pay factor is simply the product of the life cycle cost per square unit of variability measured in the as-constructed product

The quality index is probably the best method of determining the quality delivered. As in the example, the quality index is used in developing the pay factor. Assume a target quality index value that would received one hundred percent pay, say 3

$$PF = 1 - \lambda \frac{G\left(100\left(QI_T - QI_D\right)\right)^2}{C}$$

where

G = Weight factor to be determined by the agency

QI_T= Target quality index

QI_D= Quality index delivered

λ = Lot life cycle cost factor per square unit quality

C = Bid price ($)

5.5 Bonuses

Many states have included in their construction contract package some form of bonus clause. The merit of such practice has been widely discussed among highway officials, and the prevailing attitude supports or even assures its survival. Weed [1996] concluded that an incentive in the form of a bonus is cost-beneficial and is in the public interest, in addition to positive psychological benefit.

The researcher maintains that quality received is generally quality paid for and that there is an ethical merit in attaching a bonus directly to a project. The concept of a market economy where competition is the driving force should be as valid and applicable in the highway construction industry as it is for the rest of the country. There are several reasons why the researcher opposes such practice. Consider a few of them. It encourages submittal of unbalanced bids in that the contractors expect to improve their profit margin in terms of bonuses. It is almost impossible for a contractor to consistently produce a product of higher quality than expected at no cost. Consequently this added cost should have been reflected in the bid. For example, If a contractor develops a design mix capable of reaching high strength at a lower cost than his or her competitors, then this fact should only increase his or her probability of winning the next job through a solid lower bid. The construction industry should not be isolated in its business practice from the rest of the country. It is a known fact that government contracts are usually more financially rewarding when compared to similar private contracts.

The methodology presented in this dissertation and the concept of the loss function, which asserts that any deviation from target value incurs a loss to society, such loss either borne by the contractor or by the highway agency, provide no flexibility for bonuses. However, the researcher understands the influence of culture and politics on the decision maker. In conclusion, the researcher advances that if there should be any bonuses at all, there should also be a change in its governing philosophy. In a nutshell bonuses should be the result of continued high performance on the part of the contractor and not attached or related to any particular project.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

The successful development of a methodology for transportation agencies to use in determining optimum acceptance plans for Portland cement concrete pavement has brought into focus a variety of observations The remainder of this chapter is dedicated for the presentation of concise summary of the research results, as they are responsive to the research objectives stated in chapter In addition some of the observations that became evident during the course of this research, and their implication on future research are enumerated. Concluding remarks include a scrutiny of the modeling for the optimum sample size, and the modifications needed to use it for different design criteria.

6.1 Research Summary

In this section, the research reported in this dissertation is summarized. An investigation of the concept of an optimal sampling plan (optimum sample size) was undertaken. The optimum design concept demands that, not only costs are considered, but they must be integrated in the solution The classical statistical approach to sampling did not allow for explicit cost considerations and deterministic mathematical programming techniques did not address the probabilistic nature of product acceptance sampling. A bayesian statistical framework was adopted in this dissertation, for it allowed for integration of the two disciplines.

Although a significant amount of money is spent each year in sampling and testing products for acceptance and any effort at reducing this cost is justified, the research considered a broader scope. This was due to the fact that, early in the research it was recognized that the mere development of an optimum plan based solely on cost considerations (acceptance sampling costs) would be of limited use to the FHWA. While this would satisfy the stated objective, the researcher find the potential impact of such limited effort on the FHWA quality assurance program questionable. A more comprehensive look at the quality control program had to be taken in order for the true optimum plan to be developed.

The larger the sample size the lesser the risk of making a type I or type II error. Of course the law of diminishing return does play a role. The question that had to be answered, and was answered in the research is: how much is enough, how can one determine how much is optimum. The research reported that, for pavements, the cost of either type I or type II error is far greater then the cost of sampling. For this reason, the model includes the expected cost of these errors.

In formulating the cost structure, which is defined in three cost components, life cycle costs of pavement were considered. The research reported that life cycle costs are essential to properly assess the cost of maintenance, repairs and replacement that is expected to occur over time.

Sensitivity analyses show that the model is sensitive to the change in the range of acceptability, variability of the product under examination, and cost parameters.

especially the cost of rejection. The fixed and variable cost of sampling did not greatly influence the optimum sample size (increase in the total cost of Inspection were reported). The research reported that this was due to disproportionately large cost of acceptance and rejection with respect to the fixed and variable cost.

Throughout the research Taguchi's approach to quality control is evident and the merit of his methods are discussed. The quadratic loss function was also used to develop a pay-adjustment factor

6.2 Research Findings

The optimum sample size model and the approach to quality control presented in this dissertation, if adopted, will enhance the FHWA ability to develop affective and efficient quality control programs.

Quality must first and foremost be considered as a design problem. Optimum sampling plans will yield true optimum results if and only if products are design for optimum conditions.

Some key results.

1 The researcher concludes that, the design and quality of Portland cement pavement can only be properly gauged by its complete life cycle costs. That is to say, the selection of a design must not be based on first cost considerations only; rehabilitation, operation and maintenance costs are to be included.

2. The focus on achieving of quality must be an integrated in the design process. This requires the application of parameter design techniques that will assist in selecting and evaluating the quality characteristics for suitability There must be a direct correlation between the quality characteristics and the performance criteria that they are intended to represent.

3. In evaluating and deciding among design alternatives, the use of distress models of quality characteristics should be used and optimum levels should be should be sought. The selected optimum design must have the lowest life cycle cost.

4. The percent defective method was found not to be as good a yardstick to measure quality as previously thought this position taken by the researcher in this report, is supported by Dr. Genichi Taguchi's research cited in the literature search section Setting a target value for the quality characteristic and a variability control mechanism (e g., standard deviation) is a better quality measure

5. The loss in quality as a result of deviation of the mean of a particular quality characteristic from the designed target value can be evaluated using the given quadratic loss function Where λ the cost coefficient, is derived in terms of life cycle costs

$$L(y) = \lambda(\sigma^2 + \mu^2)$$

6. In this dissertation, four types of targets were identified. Each target type represents a uniquely defined function that relates the economic loss and the deviation from target value. They are as follows: large the better, smaller the better, nominal the best, and asymmetric nominal the best.

7. A cost model was developed to determine the optimum sample size. It considers all relevant cost components, namely the cost of inspection, the cost of acceptance, and the cost of rejection. The model reflects the probabilistic nature of the cost of acceptance and the cost of rejection, while the cost of inspection is represented in terms of fixed and variable costs.

The model was developed within a bayesian framework, as are most economic models. This flexibility allowed to consider prior and posterior probabilities distributions(although not necessary), which can be useful to agencies, where between lot variation is a concern. The model followed this basic structure presented by Mace in 1964.

$$\int_{-\infty}^{\infty} l(\varepsilon) f(\varepsilon, n) d\varepsilon + C(n)$$

To yield

$$N \int_{-\infty}^{\infty} l(\varepsilon) f(\varepsilon, n) d\varepsilon + C(n)$$

8. The fixed and variable cost of sampling did not dominate the determination of the sample size.

9. Inferences drawn from sensitivity analyses suggest that a fixed sample size of five, as is commonly used in practice is not sufficient to properly discriminate among good or bad concrete pavement product.

10. As oppose to risk based plans, where price adjustment must be incorporated in the plan for the assumed risk, economically based plans do not require this step to be effective. However, the researcher suggests that once the sample size and the probability of acceptance are obtained, O.C. curve could be drawn specifically to determine the remaining beta risk (only as a check). Equation 5.3 (rewritten below) should be used for pay adjustment.

$$PF = 1 - \lambda \frac{G(100(QI_T - QI_a))^2}{C}$$

6.3 Concluding Remarks

In this section some of the observations made in the course of this research is discussed. The research reports the ability of the model to determine the optimum sample size for the purpose of concrete pavement acceptance. In fact the model can only assist the engineer in reaching such decision. There are several issues that must be considered prior to reaching final decision on a particular plan. The optimum plan in terms of cost may bring unwanted risk. The engineer is advised to exercise judgment in making such decision.

6.4 Recommendations for Future Research

The researcher recommends that FHWA actively pursue modifications to current quality control philosophy. The methods and ideas suggested in this dissertation can serve as a starting point. The inclusion of a parameter-design step in their current design method is recommended. As for a continuation to this research, in particular the cost model, efforts toward refining the model to explicitly considers cases where variance unknown could produce worthwhile results.

There is a need for developing relative weight factors for each quality characteristic since the impact of each on the performance of the pavement is different. A better understanding of the influence of the compressive strength on product performance, which includes the durability of the product, should be pursued.

APPENDIX A

MATHEMATICAL SOLUTIONS

Modeling the Optimum Sampling Plan with the Larger the Better Quality Cost Function

$$\mathrm{ETC1}(n\ U) = C_I + C_A + C_R$$
$$C_I = C_s + nC$$

$$C_R = (\mathrm{N}-n)C_R - (\mathrm{N}-n)C_r \int_{-\infty}^{\infty} \mathrm{P}_a(\mu)h(\mu)d\mu$$

$$\int_{-\infty}^{\infty} \mathrm{P}_a(\mu)h(\mu)d\mu = \int_{-\infty}^{\infty}\int_{-\infty}^{L} \frac{\sqrt{n}}{\sigma\sqrt{2\pi}} e^{-\frac{(\bar{x}-\mu)^2}{2\sigma^2/n}} \frac{\sqrt{D}}{\sigma\sqrt{2\pi}} e^{-\frac{\mu^2}{2\sigma^2/D}} d\bar{x}d\mu$$

$$= \frac{\sqrt{nD}}{2\pi\sigma^2} \int_{-\infty}^{L}\int_{-\infty}^{\infty} e^{-\frac{n(\bar{x}-\mu)^2+D\mu^2}{2\sigma^2}} d\mu d\bar{x}$$
$$= \frac{\sqrt{nD}}{2\pi\sigma^2} \int_{-\infty}^{L}\int_{-\infty}^{\infty} e^{-\frac{(n+D)}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)^2 - \frac{nD}{2\sigma^2(n+D)}\bar{\mathrm{X}}^2} d\mu d\bar{x}$$
$$= \frac{\sqrt{nD}}{2\pi\sigma^2} \int_{-\infty}^{L} e^{-\frac{nD}{2\sigma^2(n+D)}\bar{\mathrm{X}}^2} \int_{-\infty}^{\infty} e^{-\frac{(n+D)}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)^2} d\mu$$

$$= \frac{\sqrt{\pi nD}}{2\pi\sigma^2} \int_{-\infty}^{L} e^{-\frac{nD}{2\sigma^2(n+D)}\bar{\mathrm{X}}^2} d\bar{x}$$

$$C_A = (\mathrm{N}-n)\lambda \int_{-\infty}^{\infty} \frac{1}{\mu^2}\left(1+\frac{3\sigma^2}{\mu^2}\right)\mathrm{P}a(\mu)h(\mu)d\mu$$

$$= (\mathrm{N}-n)\lambda \int_{-\infty}^{\infty} \frac{1}{\mu^2}\mathrm{P}a(\mu)h(\mu)d\mu + (\mathrm{N}-n)\lambda\, 3\sigma^2 \int_{-\infty}^{\infty} \frac{1}{\mu^4}\mathrm{P}a(\mu)h(\mu)d\mu$$

Given that

$$C_\lambda(\mu) = \mathrm{N}\left[\int_{-\infty}^{\infty} g(x) f(x,\mu)\right] Pa(\mu)$$

$$Pa(\mu) = \int_{L}^{\infty} g(\bar{x}\ \mu)$$

$$g(x) = \lambda \frac{1}{\mu^2}\left[1 + 3\frac{\sigma^2}{\mu^2}\right]$$

$$\int_{-\infty}^{\infty} g(x) f(x,\mu)dx = \int_{-\infty}^{\infty} \frac{1}{\mu^2}\left[1+3\frac{\sigma^2}{\mu^2}\right]\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma^2}e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}dx$$

$$\frac{1}{\mu^2}\left[1+\frac{3\sigma^2}{\mu^2}\right]$$

$$C_\lambda(\mu) = \mathrm{N}\left[\frac{1}{\mu^2}\left[1+\frac{3\sigma^2}{\mu^2}\right]\right] Pa(\mu)$$

$$= (\mathrm{N}-n)\lambda\frac{\sqrt{nD}}{2\pi\sigma^2}\int_L^{\infty}\int_{-\infty}^{\infty}\frac{1}{\mu^2}e^{-\frac{(n+D)}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)^2-\frac{nD}{2\sigma^2(n+D)}\bar{x}^2}d\mu dx$$

$$+\frac{3\lambda(\mathrm{N}-n)}{2\pi}\sqrt{nD}\int_L^{\infty}\int_{-\infty}^{\infty}\frac{1}{\mu^4}e^{-\frac{(n+D)}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)^2-\frac{nD}{2\sigma^2(n+D)}\bar{x}^2}d\mu dx$$

$$= (\mathrm{N}-n)\lambda\frac{\sqrt{nD}}{2\pi\sigma^2}\int_L^{\infty}e^{-\frac{nD}{2\sigma^2(n+D)}\bar{x}^2}\int_{-\infty}^{\infty}\frac{1}{\mu^2}e^{-\frac{(n+D)}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)^2}d\mu dx$$

$$+\frac{3\lambda(\mathrm{N}-n)}{2\pi}\sqrt{nD}\int_L^{\infty}e^{-\frac{nD}{2\sigma^2(n+D)}\bar{x}^2}\int_{-\infty}^{\infty}\frac{1}{\mu^4}e^{-\frac{(n+D)}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)^2}d\mu dx$$

It can be concluded now

$$\mathrm{ETCI}(n,\mu) = C_s + nci + \frac{\sqrt{\pi nD}}{2\pi\sigma^2}\int_L^{\infty}e^{-\frac{nD}{2\sigma^2(n+D)}\bar{x}^2}dx$$

$$+(\mathrm{N}-n)\lambda\frac{\sqrt{nD}}{2\pi\sigma^2}\int_L^{\infty}e^{-\frac{nD}{2\sigma^2(n+D)}\bar{x}^2}\int_{-\infty}^{\infty}\frac{1}{\mu^2}e^{-\frac{n+D}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu\right)}d\mu dx$$

$$+\frac{3\lambda(\mathrm{N}-n)}{2\pi}\sqrt{nD}\int_L^{\infty}e^{-\frac{nD}{2\sigma^2(n+D)}\bar{x}}\int_{-\infty}^{\infty}\frac{1}{\mu^4}e^{-\frac{n+D}{2\sigma^2}\left(\frac{n}{n+D}\bar{x}-\mu^2\right)}d\mu dx$$

Taking expectation with respect to (μ)

$$C_A = N\int_{-\infty}^{\infty}\frac{1}{\mu^2}\left[1+\frac{3\sigma^2}{\mu^2}\right]Pa(\mu)h(\mu)d\mu$$

$$= N\int_{-\infty}^{\infty}\frac{1}{\mu^2}Pa(\mu)h(\mu)d\mu + 3N\sigma^2\int\frac{1}{\mu^4}Pa(\mu)h(\mu)d\mu$$

It is known that f(x u) is normal with mean (μ) and variance(σ^2)
Thus

$$Pa(\mu) = 1-\Phi\left(\frac{L-\mu}{\sigma/\sqrt{n}}\right) = 1-\Phi\left[\frac{(L-\mu)\sqrt{n}}{\sigma}\right]$$

For standard normal distribution

$\Phi = cdf$

$$ETCA = N\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}g(x)f(x\ \mu)dx\right]h(\mu)d\mu$$

$$= N\int_{-\infty}^{\infty}\left[\frac{1}{\mu^2}\left[1+\frac{3\sigma^2}{\mu^4}\right]\int_{-\infty}^{\infty}f(x\ \mu)dx\right]h(\mu)d\mu$$

$$= N\int_{-\infty}^{\infty}\frac{1}{\mu^2}\left[1+\frac{3\sigma^2}{\mu^2}\right]h(\mu)d\mu$$

$$= \int_{-\infty}^{\infty}\frac{N}{\mu}h(\mu)d\mu + 3\sigma^2\int\frac{1}{\mu^4}h(\mu)d\mu$$

$$= N\left(\frac{D}{2\sigma^2}+3\left(\frac{3D^2}{4\sigma^3}\right)\right) - \frac{ND}{\sigma^3}\left(\frac{1}{2}+\frac{9D}{4}\right)$$

$$ETCR = NC_r$$

Now compute $\frac{\partial}{\partial L}ETCR(n\ \mu)$

$$= \frac{\partial}{\partial}\left[C + nC + (N\ \ n)C_r + (N - n)\lambda\int_{-\infty}^{\infty}\frac{1}{\mu^2}Pa(\mu)h(\mu)d\mu\right]$$

$$+3\sigma^2(N\ \ n)\lambda\int_{-\infty}^{\infty}\frac{1}{\mu^4}Pa(\mu)h(\mu)d\mu - (N - n)C_rPa$$

$$(N - n)\lambda\int_{-\infty}^{\infty}\frac{1}{\mu^2}\frac{\partial}{\partial L}(Pa(\mu))h(\mu)d\mu$$

$$+3\sigma^2(N - n)\lambda\int_{-\infty}^{\infty}\frac{1}{\mu^4}\frac{\partial}{\partial L}(Pa(\mu))h(\mu)d\mu \quad (N\ \ n)C\frac{\partial}{\partial L}Pa$$

$$\frac{\partial}{\partial L}\left(Pa(\mu)\right)=\frac{\partial}{\partial L}\int_L^{\infty} g(\bar{x},\mu)d\bar{x}=g(L,\mu)=\frac{\sqrt{n}}{\sigma\sqrt{2\pi}}e^{-\frac{(L-\mu)}{2\sigma^2/n}}$$

$$\frac{\partial}{\partial L}ETCI(n,U)=(N-n)\lambda\frac{\sqrt{nD}}{2\pi\sigma^3}\int_{-\infty}^{\infty}\frac{1}{\mu^2}e^{-\frac{n(L-\mu)^2}{2\sigma^2}-\frac{\mu^2D}{2\sigma^2}}d\mu$$

$$+3\sigma^2(N-n)\lambda\int_{-\infty}^{\infty}\frac{1}{\mu^4}e^{-\frac{n(L-\mu)^2+\mu^2D}{2\sigma^2}}d\mu$$

$$+\frac{\sqrt{\pi nD}}{2\pi\sigma^2}e^{-\frac{nDL^2}{2\sigma^2(n+D)}}>0$$

A solution to a first derivative test that is greater than zero, as it has been obtained in this case, indicated that there is no maximum or minimum point

General Solutions

$$U=\int_{-\infty}^{\infty}\mu^2 Pa(\mu)h(\mu)d\mu$$

$$=\int_{-\infty}^{\infty}\mu^2[\Phi[(U-\mu)\frac{\sqrt{n}}{\sigma}]-\Phi[(-U-\mu)\frac{\sqrt{n}}{\sigma}]]\frac{\sqrt{D}}{\sqrt{2\pi}\sigma}e^{-\frac{D\mu^2}{2\sigma^2}}$$

Consider

$$1)\int_{-\infty}^{\infty}\mu^2\Phi[(A-\mu)\frac{\sqrt{n}}{\sigma}]e^{-\frac{D}{2\sigma^2}\mu^2}\frac{\sqrt{D}}{\sqrt{2\pi}\sigma}d\mu$$

let

$$g(\mu)=\mu e^{-\frac{D\mu^2}{2\sigma^2}}$$

$$H(\mu)=\mu\Phi\left[(A-\mu)\frac{\sqrt{n}}{\sigma}\right]$$

$$=-\frac{\sigma^2}{D}e^{-\frac{D\mu^2}{2\sigma^2}}\mu\Phi\left[(A-\mu)\frac{\sqrt{n}}{\sigma}\right]\Big|_{-\infty}^{\infty}\frac{\sqrt{D}}{\sqrt{2\pi}\sigma}$$

$$+\int_{-\infty}^{\infty}\frac{\sigma^2}{D}\frac{\sqrt{D}}{\sqrt{2\pi}\sigma}e^{-\frac{D\mu^2}{2\sigma^2}}\left[\Phi\left[(A-\mu)\frac{\sqrt{n}}{\sigma}\right]+\mu e^{-\frac{(\mu-A)^2}{2\sigma^2}n}\frac{\sqrt{n}}{\sigma}d\mu\right]$$

(1) (2)

$$-\frac{\sigma^2}{D}\int_{-\infty}^{\infty}\frac{\sqrt{D}}{\sqrt{2\pi\sigma^2}}e^{-\frac{D\mu^2}{2\sigma^2}}\Phi\left[(A-\mu)\frac{\sqrt{n}}{\sigma}\right]d\mu$$

$$+\frac{\sigma^2}{D}\int_{-\infty}^{\infty}\frac{\sqrt{nD}}{\sqrt{2\pi\sigma^2}}e^{-\left[\frac{D\mu^2}{2\sigma^2}+\frac{(\mu-A)^2}{2\sigma^2}n\right]}$$

$$(1)=\frac{\sigma^2}{D}Pr(X+U\le A) \quad \text{where } X\approx N\left(0\frac{\sigma^2}{n}\right) U\approx N\left(0,\frac{\sigma^2}{D}\right)$$

$$\text{since}\quad X+U\approx N\left(0,\frac{\sigma^2}{n}+\frac{\sigma^2}{D}\right)$$

$$(1)=\frac{\sigma^2}{D}\Phi\left(\frac{A}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}\right)$$

$$(1)\int_{-\infty}^{\infty}\Phi\left[\frac{(t-\mu)}{\sigma}\sqrt{n}\right]\Phi\left(\frac{\mu}{\sigma\mu}\right)d\mu=P(X+U\le t)$$

where $X+U\approx N\left(0,\frac{\sigma^2}{n}\right)$, $U\approx N\left(0,\sigma^2\mu\right)$, X and U and

$$X+U\approx N\left(0,\frac{\sigma^2}{n}+\sigma^2\mu\right)$$

$$P(X+U\le t)=\Phi\left(\frac{t}{\sqrt{\frac{\sigma^2}{n}+\sigma^2\mu}}\right)$$

$$(2)-\frac{\sigma^2}{D}\frac{\sqrt{nD}}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty}\mu e^{-\frac{1}{2\sigma^2}\left[D\mu^2+n\mu^2-2nA\mu+nA^2\right]}d\mu$$

$$=\frac{\sqrt{n}}{\sqrt{2\pi D}}\int_{-\infty}^{\infty}\mu e^{-\frac{(n+D)}{2\sigma^2}\left[\mu^2-2\frac{nA}{n+D}\mu+\left(\frac{n}{n+D}\right)^2A^2\right]}+e^{-\frac{nDA}{2\sigma^2(n+D)}}d\mu$$

$$= \frac{\sqrt{n}}{\sqrt{2\pi D}}\frac{\sqrt{2\pi}\sigma}{\sqrt{n+D}}\left\{\int_{-\infty}^{\infty}\frac{\sqrt{n+D}}{\sqrt{2\pi}\sigma}e^{-\frac{(n+D)}{2\sigma^2}\left[\text{[illegible]}\, D^{A}\right]^2}\mu\, d\mu\right\}e^{-\frac{nDA^2}{2\sigma^2(n+D)}}$$

$$= \sqrt{\frac{n}{n+D}}\frac{\sigma}{\sqrt{D}}e^{-\frac{nDA^2}{2\sigma^2(n+D)}}$$

substitute A by U and U

For A = U $L_1 = (1) + (2) = \frac{\sigma^2}{D}\Phi\left[\frac{U}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}\right] + \sqrt{\frac{n}{n+D}}\frac{\sigma}{\sqrt{D}}e^{-\frac{nDU^2}{2\sigma^2(n+D)}}$

For A = − U $L_2 = (1) + (2) - \frac{\sigma^2}{D}\Phi\left[\frac{-U}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}\right] + \sqrt{\frac{n}{\eta+D}}\frac{\sigma}{\sqrt{D}}e^{-\frac{nDU^2}{2\sigma^2(n+D)}}$

$$L = L_1 \; L_2 - \frac{\sigma^2}{D}\left\{\Phi\left[\frac{U}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}\right] - \Phi\left[\frac{-U}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}\right]\right\}$$

$$\Phi(X) - \int_{-\infty}^{X} e^{-\frac{y^2}{2}}\frac{1}{\sqrt{2\pi}}dy$$

$$\int_{-\infty}^{\infty}\mu^2 Pa(\mu)h(\mu)d\mu$$

$$= \frac{\sigma^2}{D}\left\{\int_{-\infty}^{\frac{\mu}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy \quad \int_{a}^{\frac{\mu}{\sqrt{\frac{\sigma^2}{D}+\frac{\sigma^2}{n}}}}\frac{1}{\sqrt{2\pi}}dy\right\}$$

$$\frac{\sigma^2}{D}Pa$$

where

$$g(U|\mu) = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(U-\mu)^2}{2\sigma^2/n}\right]$$

$$h(\mu) = \frac{\sqrt{D}}{\sigma\sqrt{2\pi}} \exp\left[-\frac{\mu^2}{2\sigma^2/D}\right]$$

$$g(U|\mu)h(\mu) = \frac{\sqrt{nD}}{2\pi\sigma^2} \exp\{-(1/2)\left[\frac{nU^2 - 2nU\mu + n\mu^2 + D\mu^2}{\sigma^2}\right]\}$$

$$= \frac{\sqrt{nD}}{2\pi\sigma^2} \exp\left\{-(1/2)\left[\frac{\{\mu - [nU/(n+D)]\}^2}{\sigma^2/(n+D)} + \frac{nU^2(n+D) - n^2U^2}{\sigma^2(n+D)}\right]\right\}$$

$$= \frac{\sqrt{nD}}{2\pi\sigma^2} \exp\left\{-(1/2)\left[\frac{(\mu - y)^2}{\tau^2} + \frac{nDU^2}{\sigma^2(n+D)}\right]\right\}$$

$y = nU/(n+D)$,

$$\tau = \sigma/\sqrt{n+D}$$

$$g(-U|\mu)h(\mu) = \psi(U)\omega-(\mu)$$

$$g(U|\mu)h(\mu) = \psi(U)\omega+(\mu)$$

$$g(U|\mu)h(\mu) = \frac{1}{\tau\sqrt{2\pi}} \exp\left[-\frac{(\mu - y)^2}{2\tau^2}\right] \cdot \frac{\sqrt{nD/(n+D)}}{\sigma\sqrt{2\pi}} \exp\left[-\frac{nDU^2}{2(n+D)\sigma^2}\right]$$

$$I = \int_{-\infty}^{\infty} \mu^2 [g(U|\mu) + g(-U|\mu)] h(\mu) d\mu$$

$$I_1 = \int_{-\infty}^{\infty} \mu^2 g(U|\mu) h(\mu) d\mu + \int_{-\infty}^{\infty} \mu^2 g(-U|\mu) h(\mu) d\mu$$

$$= \int_{-\infty}^{\infty} \mu^2 \psi(U) \omega+(\mu) d\mu + \int_{-\infty}^{\infty} \mu^2 \psi(U) \omega-(\mu) d\mu$$

$$= \psi(U)\left[y^2 + \tau^2\right] + \psi(U)\left[(-y^2) + \tau^2\right]$$

$$= 2\psi(U)\left[\frac{n^2U^2}{(n+D)^2} - \frac{\sigma^2}{n+D}\right]$$

$$I_1 = \int_{-\infty}^{\infty} \left[g(U|\mu) + g(-U|\mu)\right] h(\mu) d\mu$$

$$I_2 = \int_{-\infty}^{\infty} g(U|\mu) h(\mu) d\mu + \int_{-\infty}^{\infty} g(-U|\mu) h(\mu) d\mu$$

$$= \int_{-\infty}^{\infty} \psi(U) \omega+(\mu) d\mu + \int_{-\infty}^{\infty} \psi(U) \omega-(\mu) d\mu$$

$$= 2\psi(U)$$

For the one-sided case

$$I_1 = \psi(U)\left[\frac{n^2U^2}{(n+D)^2} - \frac{\sigma^2}{n+D}\right]$$

$$I_2 = \psi(U)$$

Mathematical and statistical solutions were checked by
Mahfoud Ziad, Ph.D student Department of Statistics
Omar de La Cruz, Ph.D candidate, Department of Mathematics
Chen Ping Wang Ph.D student, Department of Statistics

MathCAD Input-Output example

$cs \quad 200 \qquad D \quad 5 \qquad k \quad 2 \qquad e \quad 300 \qquad\qquad N \quad 1 \qquad cr \quad 300000$

$n \quad 1\ 2\ 20 \qquad\qquad cl \quad 15$

$$u(n) \quad \sqrt{\left|cr\,(n \quad D) \quad (n + D + 1)\,k{\cdot}e^{2}\right| \left(\left(\frac{n+D}{k{\cdot}n^{2}}\right)\right)} - \frac{1}{2}\cdot\sqrt{2}\,\sqrt{(120000\,n \quad 420000)\,\frac{(n+5)}{n^{2}}}$$

u(n)
1272.792
759.934
588.784
503.115
451.664
417.333
392.792
374.374
360.041
348.580
339.177
331.348
324.721
319.039
314.113
309.801
305.995
302.612
299.584
296.859

$$[illegible]\ \frac{k\,c}{D} \left| \int_{\infty} \frac{n}{D} \; e^{y^2} \frac{dy}{2x} \right| \int_{\infty} \frac{n}{D} \frac{1\,e^{\frac{y^2}{2}}}{\sqrt{2x}}\,dy \right] \rightarrow 390200 + 15\,n \quad 64000\text{-}\omega\left[\frac{1}{6}\sqrt{6}\,\sqrt{(2\,n+7)\ [illegible]}\right.$$

216224.031
216417.029
216849.855
217303.224
217727.384
218104.341
218434.175
218722.21
218974.645
219197.[illegible]68
219394.018
219570.999
219779.585
219873.654
220003.599
220123.025
220232.83
220334.250
220428.357
220516.095

$$1 \int_{-\infty}^{\frac{u(n)}{\sqrt{\left(\frac{n^2}{D}\quad\frac{n}{n}\right)}}} \frac{1 e^{\frac{y^2}{2}}}{\sqrt{2\pi}}\,dy \to \frac{1}{2}+\frac{1}{2}\operatorname{erf}\left[\frac{1}{6}\sqrt{6}\,\sqrt{(2n+7)\frac{(n+5)}{n^3}}\quad\frac{\sqrt{5}}{\sqrt{\frac{(n+5)}{n}}}\right]$$

1
0.999
0.996
0.994
0.991
0.989
0.987
0.986
0.984
0.983
0.982
0.981
0.98
0.979
0.979
0.978
0.978
0.977
0.977
0.976

$$u(\infty) \quad \frac{\sigma}{D} \quad \int \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\,dy \qquad \frac{u(n)}{\sqrt{\left(\frac{\sigma^2}{D} - \frac{\sigma^2}{n}\right)}} \quad \int \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\,dy \quad \operatorname{erf}\left(\frac{1}{\theta}\sqrt{\theta}\,\sqrt{(2n+7)\,\frac{(n+5)}{n^2}}\right)$$

$$= \begin{array}{|c|}
1 \\
0.998 \\
0.993 \\
0.988 \\
0.983 \\
0.978 \\
0.975 \\
0.971 \\
0.969 \\
0.966 \\
0.964 \\
0.962 \\
0.96 \\
0.959 \\
0.957 \\
0.956 \\
0.955 \\
0.954 \\
0.953 \\
0.952
\end{array}$$

$$m + (N)\cdot\sigma \quad d\,n \quad (N)\left(k\cdot\sigma^2 + \frac{k\cdot\sigma^2}{D} \quad \sigma\right)\left| 1 \quad \int_{c}^{\frac{u(n)}{\sqrt{\left(\frac{\sigma^2}{D}+\frac{\sigma^2}{n}\right)}}} \frac{1\ e^{\frac{y^2}{2}}}{\sqrt{2\pi}}\,dy\right| \rightarrow 258200 + 15\cdot n \quad 42000\cdot\mathrm{erf}\left[\frac{1}{6}\sqrt{6}\sqrt{(2n+7)\frac{(n\quad 5)}{n^2}}\quad\frac{\sqrt{5}}{\sqrt{\frac{(n+5)}{n}}}\right]$$

216219.55
216333.54
216547.427
216781.612
217001.192
217197.171
217369.587
217521.105
217654.823
217773.584
217879.809
217975.5
218062.293
218141.527
218214.299
218281.513
218347.9[illegible]
218402.[illegible]
218456.079
218508.003

APPENDIX B
NOTES ON PARAMETER AND TOLERANCE DESIGN

In this section, emphasis is placed on a very important concept, upon which rest the foundation of this dissertation, referred to as quality engineering. Earlier, the researcher stated that it was necessary to establish a framework within which development of optimal pavement design can be made possible. The achievement of optimal pavement design will in turn justify the need for optimum acceptance plans. Quality engineering form the view of Dr Taguchi provides such a vehicle.

Also this chapter concerns probably the largest contribution to quality methodology that Taguchi has made, according to Phillip J Ross [1996]. Literatures have shown that designed experiments have existed since the 1930s. However, previous experimental approaches looked upon all factors as causes of variation. If these causes could be well controlled or eliminated, then product or process variation would be reduced, and, therefore, quality would be improved. This is the approach suggested by Willenbrock [1976] in his manual for statistical quality control for construction highway. The researcher believes that, although it is very important to quantify product variabilities and their sources; the method of response and dealing with them is even more important. If for example, a product is sensitive to ambient temperature variations, how can anyone control or eliminate the weather conditions in a customer's environment? The answer is obvious: ambient temperature variations can neither be controlled nor eliminated without a large expense. This example is analogous to the situation that transportation agencies face in designing pavement to be exposed to severe weather condition. The researcher support Philip J Ross conclusion that ambient temperature in a factory setting, might be controlled very well to eliminate the temperature affect on a machine s performance, but the world s atmosphere cannot be controlled so all kinds of devices are exposed to large temperature variations Therefore, reason demands that a different approach be taken if product quality is to be improved Taguchi [1986] called this approach parameter design. Its objective is to improve quality without controlling or eliminating causes of variation This is a fundamental departure from the current philosophy in vigor in the highway construction industry Taguchi believes that attempt at controlling or eliminating causes of variation is very expensive compared to a parameter design approach.

A Quality Engineering Approach

In Bendell's [1989] book title Taguchi Methods which is a collection of several papers written by different authors, Manton S M [1989] one of the authors pointed out that although the concepts discussed in this chapter are easily understood, but some have found them difficult to apply particularly tolerance design. The researcher believes that, in the case of highway construction, these techniques can be applied with ease

Taguchi as was explained earlier in this research, views the design of a product or process as a three-phase program

1 System design

2. Parameter design

3 Tolerance design

Figure B 1 shows these steps in a flow chart for quality engineering optimization. It is this complete and integrated cycle focus on achieving quality that the researcher is proposing in this research In the System design phase new concepts, ideas, methods, etc , are generated to provide new or improved products to customers. An important point that Taguchi made, and the researcher wish to direct special attention to is that the parameter design phase is crucial to improving the uniformity of a product and can be done at no cost or even at a savings This means that certain parameters or functional characteristics of a product or process design are set to make the performance less sensitive to causes of variation, that is, these functional characteristics are set at optimal levels The tolerance design phase focuses on improving quality at a minimal cost. Tightening tolerances on product or process parameters to reduce the performance variations does this. It is normally done after the parameter design It is to be noted that parameter design is this very important step that current methodology lacks

Typically, when a problem is detected in product development, an engineer may jump directly to tolerance design, when tolerances are tightened, variation will be reduced and quality improved However, tightening tolerances may be expensive and completely unnecessary if parameter design were used first. Taguchi cautioned designers from making the serious mistake to use expensive materials, components, or processes for a product when lower cost items may be used if a parameter design approach is applied.

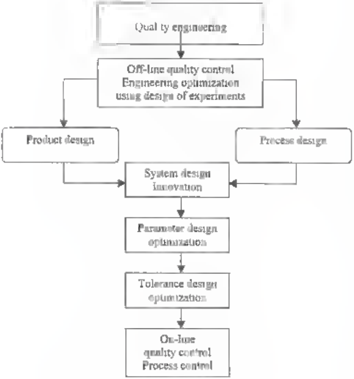

Quality engineering
Off-line quality control
Engineering optimization
using design of experiments
Product design
Process design
System design
innovation
Parameter design
optimization
Tolerance design
optimization
On-line
quality control
Process control

Figure B 1 The three steps involved in engineering optimization

Control and Noise Factors

To begin the discussion on parameter design, one must consider types of design and development factors. Taguchi separates factors into two main groups: control factors and noise factors. Control factors are those, which are set by the manufacturer and cannot be directly changed by the customer. It was noted earlier in this research that the contractor-client relationship that exists in the highway industry is somewhat different from the one between manufacturer-consumer in other industries. The researcher maintains that the product design process in both cases is fundamentally the same. An engine manufacturer can dictate the material for the pistons, the tension on the piston rings, the piston-bore clearance, etc., which cannot be easily modified by the customer. Noise factors are those over which the manufacturer has no direct control but which vary with the customer's environment and usage. In general, noise factors are among those that the manufacturer desires not to have to control at all.

Noise factors can be placed in three categories:

1. Outer noise
2. Inner noise
3. Between product noise

Outer noises are environmental factors such as ambient temperature, humidity, pressure, or people. Even different batches of materials may be viewed as outer noise to a production process. Inner noises are function and time-related, such as deterioration, wear, fade of color, shrinkage, and drying out. Outer noises produce variation from outside the product, inner noises produce variation from inside or within the product, and product noise manifests itself in part-to-part variation. Taguchi indicated that products may have sensitivity to all three forms of noise simultaneously The design quality of a product or process provides less functional variation due to outer or inner noise. Production quality provides less functional variation from one part to another and close to a target value Taguchi refers to design quality efforts as offline quality control and production quality efforts as on line quality control Figure B.2 top shows types of noises discussed here, and Figure B 2 bottom shows the quality control mechanism that deals with them. Off-line quality control is any of the design and development activities that may take place before products are manufactured and available to customers. When products are manufactured to be available for customers, the on line activities begin Of course, the on line activity must be well planned before the start of production. The loss function reflects the off-and on-line quality control efforts in the equation discussed in chapter 3 and 4

$$L(y) = k[S_y^2 + (y-m)^2]$$

The variance S_y^2 is reduced and the target value m determined during the offline phase. The average value produced y is controlled during the on-line phase Both of these phases will reduce the loss associated with the product. The real purpose of a quality control chart is to provide the proper close-to-the-target, average value from a process Incidental or special causes of variation may be identified and controlled or eliminated but this is the tolerance design approach. If a large amount of variation is present at the introduction of a process, this is an indication of the lack of off line quality control efforts The more off-line quality control work done, the more robust a process or product is against disturbances in the environment and life of a product Again, to reduce loss to society in the loss function there must be reduced variance (off-line quality control work) and an average near the target value (on-line quality control work).

Figure B.3 shows the contribution of noises factors to deviation of product functional characteristics from target value. Parameter design is used to dampen the effect of noise (reduce variance) by choosing the proper or optimal level for control factors. Parameter design is used to improve quality without controlling or removing the cause of variation, to make the product robust, that is, insensitive against noise factors. Taguchi uses tolerance design is used to reduce the variation by tightening tolerances around the chosen target value of the control factors. Tolerance design reduces or eliminates the effect of causes of variation. By using parameter and tolerance design, the true critical characteristics (control factors) can be identified and minimized in number. Because of the principles of parameter and tolerance design, major emphasis is placed on true control factors and very little emphasis on true noise factors. Primarily, noise factors are used in experiments to expose the robust levels of the control factors.

Product design		Process design
	Outer noise	
Consumer's usage condition Low temperature High temperature Vibration Humidity		Ambient temperature Humidity Seasons Incoming material variation Operators Batch to batch variation
	Inner noise	
Deterioration of parts Deterioration of materials Oxidation		Machinery aging Tool wear Deterioration
	Between product	
Piece to piece variation where they are supposed to be the same		Process-to-process variation where they are supposed to be the same
	Controllable factors	
All design parameters (dimensions, material and configuration)		All process design parameters All process setting parameters

Product realization	Quality control activity	Ability to reduce effect of noise factors		
		External	Internal	Unit-to unit
Product Design	System Parameter Tolerance	✓ ✓ ✓	✓ ✓ ✓	✓ ✓ ✓
Production Process Design	System Parameter Tolerance	X X X	X X X	✓ ✓ ✓
Fabrication	Detection/correction Feedforward Control Screening	X X X	X X X	✓ ✓ ✓
Customer usage	Warranty and repair	X	X	X

Figure B.2 Noise factors in products design

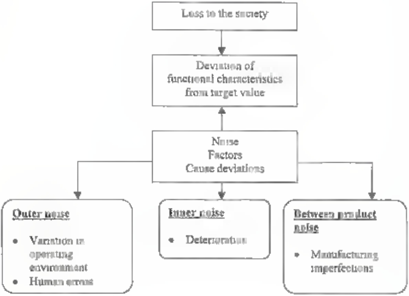

Loss to the society
Deviation of functional characteristics from target value
Noise Factors Cause deviations
Outer noise
Variation in operating environment
Human errors
Inner noise
Deterioration
Between product noise
Manufacturing imperfections

Figure B 3 Contribution of noise factors to deviation of product functional characteristics from target value

The researcher is of the opinion that the highway industry would greatly benefit if it would explore further these ideas and methods discussed here.

In traditional statistical terms, control and noise factors are also generally categorized as fixed and random factors, respectively. Fixed factors are those that may have specific values selected for their levels in a designed experiment, while random factors are those that have levels which are randomly determined, such as different operators, different machines, etc. The parameter design concept still applies however the factors are described.

It is believed that parameter design is not a difficult concept to understand or apply. Although some have found tolerance design difficult to apply, as noted earlier, the nature of the highway industry makes it suitable for this type of technique. The highway engineer, most likely just needs to know how to perform parameter and tolerance design in order to obtain the benefits.

Objective Functions in Robust design and Signal to Noise Ratio

In this section, a brief discussion on signal to noise ratio and objective functions in robust design is presented. These are important concepts and methods in parameter design that deserve a great deal of attention, however this is not possible within the confines of this dissertation.

In many experiments the quality characteristic has a specific value to be optimized. Attaining nominal thickness or minimizing the pavement surface roughness requires the study of static functions. Static functions are developed only for the larger the better, the smaller the better, nominal the best, percent defective quality characteristics. Taguchi has extended the audio concept of signal to noise ratio to experiments involving many factors. Such experiments are frequently called multi-factors experiments. The formulae for signal to noise ratio are design so that an experimenter can always select the largest factor level setting to optimize the quality characteristic of an experiment. Therefore the method of calculating the signal to noise ratio depends on whether the quality characteristic has a larger the better the smaller the better, nominal the best response. Figure B.4 shows a diagram of the factors affecting a quality characteristic

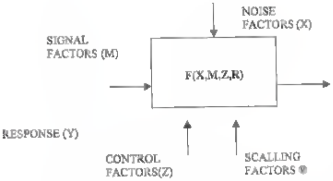


Figure B.4 Factor affecting a quality characteristics

The importance of a multi factor experiment in terms of robust design is to minimize the sensitivity of the quality characteristic to noise factors. Such decision must be based on the mean square deviation

$$MSD = \sigma^2 + (\bar{y} - m)^2$$

Such a measurement without adjusting the mean value($\bar{y}$) to the target value(m) is influenced not only by the sensitivity to noise (σ^2) but by the deviation ($\bar{y}$ m) on the target value Consequently one risks the possibility of not choosing the factors level that minimizes sensitivity to noise.

Taguchi provide a mathematical model for robust design experiment with the various parameters affecting the response as shown in figure B.4

$$y = f(X\ M\ Z, R) \qquad \text{(B 1)}$$

where y the response variable is a function of (noise) X, (signal) M, (control) Z, and (scaling) R. In reality the function may be regarded as consisting of two parts that was defined earlier as the unpredictable part and the unpredictable part

The predictable part g (M,Z,R) is referred to as the signal, and the unpredictable part *e* (X,M Z,R)

So the equation can be rewritten as

$$y = g(M, Z, R) + e(X, M, Z, R) \qquad \text{(B 2)}$$

When all the linear relationship is desired between y and M g(M,Z,R) must be a linear function

with all the non linear effects included in *e* The effect of all the noise factors is included in e(X M,Z,R). The objective of robust design is to

$$\eta = \frac{signal}{noise}$$

$$= \frac{\text{variance of the predictable part}}{\text{variance of the unpredictable part}}$$

$$= \frac{V_s}{V_e}$$

maximizes the predictable part and minimize the unpredictable part. Taguchi suggests that quality can be evaluated by the definition given.

By analogy with the communication theory η may be taken in decibel scale:

$$\eta = 10\log_{10}\left[\frac{\text{variance of the predictable part}}{\text{variance of the unpredictable part}}\right]$$

$$= 10\log_{10}\left[\frac{V_s}{V_e}\right]$$

Expressed in this way the predictable part is the signal and the unpredictable part is the noise , hence the equation is called the signal to noise ratio. It should be remembered that the objective of robust design is to maximizes the predictable part and minimize the unpredictable part. The best way to do this is by selecting suitable levels of Z and R is so that the signal to noise ratio is clearly a function of both Z and R and

$$\eta(Z,R) = 10\log_{10}\left[\frac{V_s(Z,R)}{V_e(Z,R)}\right] \qquad (B.4)$$

However, R is a factor that is used to change the overall mean effect of the response and hence the predictable part. Therefore it its not necessary to perform experiments with various values of R of R so that

$$\eta(Z) = \max \eta(Z, R\,\text{constant}) \qquad (B.5)$$

The maximum value of η may then be attained by searching through its values, usually by a set of noise, signal, control and scaling factors assigned to one or more orthogonal arrays. The optimization strategy is therefore to measure the signal to noise ratio of the response for selected values of the control (and scaling) factors by emulating the noise factors in the desired range of the signal factor. Upon evaluating η(Z) for all the control factors, the effect of each of the control factors (Z_i) on η is established so that the corresponding optimum level of Z_i may be determined to form the set of optimum control factor levels Z_0. The next step is to perform a confirmation experiment at Z_0.

When the signal factor is constant, the problem reduces to one of a static function and the signal to-noise ratio can be defined as

$$\eta = -10\log_{10}[MSD] \qquad (B.6)$$

where MSD is the mean squared deviation from target.

The signal to noise ratio also has a definite advantage over the mean squared deviation. When the target value is changed, the optimum conditions obtained by maximizing the signal to noise ratio will still be valid except for adjustment of the mean

However the same cannot be said of using the mean squared deviation from target as the objective function The optimization would have to be performed again

For nominal-the-best characteristics, the problem of minimizing the variance while keeping the mean on target is a problem of constrained optimization. By using the signal to noise ratio, the problem can be converted into an unconstrained optimization problem that is much easier to solve. The property of unconstrained optimization is the basis for one's ability to separate the actions for minimizing sensitivity to noise factors and the adjustment of the mean on target The signal to noise ratio is thus a very useful way of evaluating the quality of a process or product. This ratio measures the level of performance against the level of noise factors on performance. It is an evaluation of the stability of the performance of an output characteristic

Better performance, as measured by a high signal to noise ratio, implies a smaller loss than that measured by the corresponding quality loss function Like the quality loss function, the signal to noise ratio is an objective function of quality that takes both the effects of the mean and the variation into account Finding the correct objective function to maximize in an engineering design problem is very important. Failure to do so can lead to inefficiencies in experimentation and even wrong conclusions about optimum levels. The task of finding what adjustments are meaningful in a particular problem and determining the correct objective function (or the signal to noise ratio) is not always easy The signal to noise ratios for two functional characteristics smaller-the better and larger the better are developed below

The Signal to Noise Ratio for Smaller-the-Better

Here, the quality characteristic is continuous and non-negative, that is it can take any value from 0 to ∞ Its most desired value is zero These problems are characterized by the absence of a scaling factor or any other adjustment factor Surface roughness is an example of this type of problem

Another example of a smaller-the better type problem is the pollution from a power plant. We can reduce the total pollutants emitted by reducing the power output of the plant. However, reducing pollution by reducing power consumption does not signify any quality improvement for the power plant. Hence it is inappropriate to think of the power output as an adjustment factor In fact, the pollution per megawatt-hour of power output as the quality characteristic to be improved should be considered instead of the pollution itself

Additional examples of smaller-the-better type problems are electromagnetic radiation from telecommunications equipment, leakage current in integrated circuits and corrosion of metals Because there is no adjustment factor in these problems, the quality loss without adjustment should simply be minimized, that is minimize

$$Loss = \lambda[MSD]$$

$$= \lambda\left[\frac{1}{n}\sum_{i=1}^{n} y_i^{2}\right] \qquad (B\ 7)$$

Minimizing quality loss equivalent to maximizing the signal to noise ratio η defined by the equation (B 8) Note that η does not depend on the cost coefficient λ Also since the signal factor is a constant, and the target value is equal to zero. η merely measures the effect of noise.

$$\begin{aligned}\eta &= -10\log_{10}[MSD] \\ &= -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right] \\ &= -10\log_{10}[\sigma^2 + \bar{y}^2] \qquad \text{(B.8)}\end{aligned}$$

Signal to Noise Ratio for Larger-the-Better

Here again, the quality characteristic is continuous and non-negative, that is, it can take any value from 0 to ∞. Its target value is non-zero and, ideally, as large as possible. These problems are also characterized by the absence of a scaling factor or any other adjustment factor. Examples of such problems are the mechanical strength of a wire per unit cross-section area and the fuel efficiency of an automobile. These problems can be transformed into smaller-the-better type problems by considering the reciprocal (inverse) of the quality characteristic. That is, if y is a larger-the-better

$$\begin{aligned}\eta &= -10\log_{10}[MSD] \\ &= -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right] \qquad \text{(B.9)}\end{aligned}$$

characteristic, then taking the reciprocal (1/y) transforms the quality characteristic into a smaller the-better characteristic The objective function to be maximized in this case is given by

And the corresponding loss to society is

$$Loss = \lambda[MSD]$$

$$= \lambda\left[\frac{1}{n}\sum_{i=1}^{n} y_i^{2}\right] \qquad \text{(B 10)}$$

Certainly there exists a relationship between the signal to noise ratio and the quality loss So when comparing the loss to society at the optimum condition to the existing condition one needs to compare only the loss per unit at the optimum with the loss per unit at the existing condition This is because the loss to society is the aggregate loss of the loss per unit and can be calculated from the product of the loss per unit and the total number of units The difference in loss per unit between the optimum condition and the existing condition is the gain in loss per piece

Concrete Strength a Functional Characteristic

This brief discussion on concrete strength in this section is important because it is the strength of concrete of concrete that is used to several performance characteristics of Portland cement concrete Even though the direct relationship between and the durability and strength of concrete has not been established This conclusion is support by Wood [1996] in his manual title "Managing for Quality" Strength of concrete is commonly considered to be its most valuable property although in many practical cases other characteristics such as durability, permeability and volume stability may in fact be more important. However, strength usually gives an overall picture of the quality of the concrete because it is directly related to the structure of the cement paste.

It is also shown that concrete mixtures with equal compressive strengths do not necessarily produce equal levels of permeability, especially when *fly ash and silica fume* are included in the mixture. This lack of correlation continues until the strength reaches about 8,000 psi. At and beyond 8,000 psi, a well-defined trend is observed.

Durability is better evaluated from permeability and strength tests than from a strength test alone. Resistance to freezing and thawing is another indication of durability. However a freeze-thaw test of concrete it is not considered in Florida. Bloomquist and Armaghani [1994] in their research on concrete Durability in Florida, have concluded that Durability is greatly influenced by concrete permeability. Many mixtures designed with widely different materials and mixture proportions, can produce concrete with equal strength but different permeability levels. Concrete that meets only the strength requirement may fail to develop the expected durability if the permeability is high. The goal is to develop specifications for testing and classifying durability of concrete using data related to aggressiveness of the environments. Thus when structure in aggressive environments (such as sea water) are considered, emphasis is placed not only on achieving a concrete with optimum strength but also, and more important, on ensuring a highly durable concrete. In these situations testing for and evaluating concrete durability become equally and sometimes more important than testing for strength the quality loss function introduced in this chapter is discussed in detail in the next chapter. The assumptions, which underlie its development, are important to the extent that they enhance understanding of the intended purpose of such loss function

APPENDIX C

CONCRETE COMPRESSIVE STRENGTH DATA

Table C 1 Portland cement concrete strength data
Courtesy of New Jersey Department of Transportation

PROJECT	DATE	LOT	CLASS	N	AVG	STDV	MIN	PD
207--5V 6Q 7J	9/10/97	73A	A(4600)	6	5706	470	5290	0.00
"	10/31/97	79A	A	6	6016	420	5585	0.00
"	11/19/97	83A	A	2	5645	566	5245	0.00
"	1/21/97	84A	A	2	6105	184	5975	0.00
"	3/6/97	36A	A	1	6055	300	6055	0.00
"	3/7/97	37A	A	1	7165	300	7165	0.00
"	3/11/97	38A	A	1	6455	300	6455	0.00
"	3/12/97	39A	A	3	6870	172	6715	0.00
"	3/19/97	40A	A	1	6590	300	6590	0.00
"	3/20/97	41A	A	1	6240	300	6240	0.00
"	05//07/97	49A	A	6	4965	208	4585	8.85
"	6/5/97	67A	A	6	4955	237	4725	4.89
"	5/28/97	55A	A	1	4970	300	4970	0.00
"	5/29/97	56A	A	1	4450	300	4450	100.00
"	5/30/97	57A	A	1	5305	300	5305	0.00
"	6/3/97	58A	A	1	5475	300	5475	0.00
"	6/4/97	58A	A	1	5935	300	5935	0.00
"	6/27/97	70A	A	2	5465	665	4995	6.33
"	6/28/97	71A	A	1	6285	300	6285	0.00
"	6/29/97	72A	A	1	6005	300	6005	0.00
"	9/19/97	75A	A	3	4960	486	4510	27.04
"	8/24/96	3A	A	1	4095	300	4095	100.00
"	12/3/96	30A	A	5	5811	610	4580	1.20
"	5/14/97	51A	A	6	5585	300	5080	0.00

PROJECT	DATE	LOT	CLASS	N	AVG	STDV	MIN	PD
"	7/15/97	84A	A	6	4521	406	3985	57.11
"	4/3/97	43A	A	1	5460	300	5460	0.00
"	4/4/97	44A	A	1	5000	300	5000	0.00
"	4/7/97	45A	A	1	5925	300	5925	0.00
"	6/9/97	60A	A	1	5005	300	5005	0.00
"	6/10/97	61A	A	1	4155	300	4155	100.00
"	6/11/97	62A	A	1	4450	300	4450	100.00
"	8/6/97	65A	A	1	5390	300	5390	0.00
"	8/7/97	68A	A	1	4690	300	4690	19.39
"	4/16/97	46A	A	6	5386	230	5045	0.00
"	5/24/97	53A	A	6	4978	169	4760	0.04
"	5/14/97	51A	A	6	5569	300	5060	0.00
"	5/13/97	52A	A	1	5535	300	5535	0.00
"	5/15/97	54A	A	1	5010	300	5010	0.00
"	7/22/97	65A	A	3	5626	1017	4765	16.00
"	7/23/97	66A	A	1	5245	300	5245	0.00
"	12/13/98	32A	A	1	6055	300	6055	0.00
"	12/16/98	33A	A	2	5878	661	5410	0.00
29 & 195 –10H	7/20/95	7A	A	5	5107	361	4930	5.76
"	7/25/95	5A	A	1	5355	300	5355	0.00
"	7/27/95	11A	A	1	5345	300	5345	0.00
"	10/17/95	50A	A	4	4679	459	4230	0.00
"	10/23/97	65A	A	1	4355	300	4355	100.00
"	10/30/95	72A	A	4	5779	176	5525	0.00
"	10/31/95	73A	A	7	5318	286	4920	0.00
"	11/1/95	74A	A	2	5110	184	4980	0.00
"	11/2/95	75A	A	1	4745	300	4745	0.00
"	11/3/95	76A	A	2	5180	665	4720	20.21
"	11/6/95	77A	A	3	5357	216	5168	0.00
"	11/7/95	78A	A	1	5615	300	5615	0.00
"	11/8/95	80A	A	4	5266	166	5140	0.00
"	11/9/95	82A	A	1	5345	300	5345	0.00

PROJECT	DATE	LOT	CLASS	N	AVG	STDV	MIN	PD
"	11/10/95	83A	A	1	5480	300	5480	0.00
"	11/11/95	84A	A	1	4720	300	4720	9.18
"	11/13/95	86A	A	6	5503	311	5050	0.00
"	11/15/95	87A	A	3	5240	118	5130	0.00
"	11/16/95	88A	A	1	5400	300	5400	0.00
"	11/17/95	88A	A	5	5475	143	5360	0.00
"	11/17/95	89A	A	1	4965	300	4965	0.00
"	11/18/95	90A	A	6	5147	225	4790	0.00
"	11/21/95	91A	A	2	5478	187	5348	0.00
"	11/24/95	93A	A	2	5203	255	5018	0.00
"	11/28/95	95A	A	6	5432	151	5185	0.00
"	12/1/95	97A	A	6	5391	265	5075	0.00
"	12/2/95	98A	A	6	5413	238	5070	0.00
"	12/6/95	100A	A	6	5776	368	5260	0.00
"	12/7/95	101A	A	1	5745	300	5745	0.00
"	12/8/95	102A	A	4	6050	258	5605	0.00
"	12/13/95	103A	A	6	5446	284	5105	0.00
"	12/16/95	105A	A	6	5623	167	5405	0.00
"	7/12/95	2A	A	3	4575	216	4370	53.20
"	7/14/95	3A	A	3	4108	168	3990	100.00
"	10/9/95	52A	A	1	3810	300	3810	100.00
"	10/23/95	64A	A	1	4070	300	4070	100.00
"	7/18/95	5A	A	1	4900	300	4900	0.00
	7/20/95	6A	A	1	4900	300	4900	0.00
	8/10/95	16A	A	6	5244	77	5130	0.00
	8/17/95	25A	A	6	5126	667	3886	22.88
29 & 195—10H	8/24/95	26A	A	6	5327	272	4995	0.00
"	08/28/95	28A	A	6	5216	500	4460	10.18
"	9/30/95	46A	A	3	5480	416	5005	0.00
"	10/2/95	47A	A	1	4865	300	4865	0.00
"	10/3/95	48A	A	1	5275	300	4968	0.00
"	10/4/95	49A	A	1	5180	300	5160	0.00

PROJECT	DATE	LOT	CLASS	N	AVG	STDV	MIN	PD
▲	10/6/95	50A	A	3	5657	242	5380	0.00
▲	10/10/95	53A	A	3	5595	108	5475	0.00
	10/11/95	54A	A	2	4515	38	4490	100.00
■	10/12/95	55A	A	6	5080	224	4730	0.00
■	10/12/95	56A	A	1	5125	300	5125	0.00
■	10/13/95	57A	A	4	4994	304	4570	6.76
■	10/14/95	58A	A	1	5395	300	5395	0.00
■	10/16/95	59A	A	6	6238	300	5220	0.00
■	10/17/95	60A	A	4	4679	488	4230	44.62
■	10/18/95	61A	A	1	4490	300	4490	90.82
■	10/20/95	63A	A	7	4810	254	4485	21.03
■	10/27/95	68A	A	6	5071	326	4470	5.80
■	10/27/95	68A	A	3	5327	372	4920	0.00
▲	9/27/95	44A	A	6	5306	184	5060	0.00
▲	9/27/95	44A	A	1	4660	300	4660	29.58
▲	9/28/95	45A	A	6	5758	143	5570	0.00
▲	11/22/95	92A	A	6	5603	415	5315	0.00
▲	11/25/95	94A	A	8	5463	362	4695	0.00
▲	11/30/95	98A	A	1	5700	300	5700	0.00
▲	12/2/95	99A	A	1	5145	300	5145	0.00
▲	9/16/95	24A	A	1	3835	300	3835	100.00
▲	9/17/95	26A	A	5	5126	667	3985	22.86

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BIOGRAPHICAL SKETCH

I was born in Port-au-Prince, Haiti. I have been living in the USA since 1970. Following a five and half year tour in the US marines corps, I attended the Milwaukee School of Engineering and earned a BS in 1983. Prior to my return to academia, I worked in structural design, building construction, and prestressed structural components manufacturing.

I would like to pursue a career in research and teaching in engineering.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Fazl T Najafi
Fazil Najafi, Chairman
Associate Professor of
Civil Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Fernando Fagundo
Professor of Civil Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Peter A Kopac
Peter Kopac
Research Engineer
Federal Highway Administration

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Paul Thompson
Paul Thompson
Professor of Civil Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Elroy Bolduc
Professor, Foundations of Education

This dissertation was submitted to the Graduate faculty of the College of Engineering and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August 1998

Winfred M. Phillips
Dean, College of Engineering

Karen A. Holbrook
Dean, Graduate School